8(a)

ATOMS

CHAPTER OVERVIEW

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8(a).1. INTRODUCTION

The structure of matter that shapes the world around us has been a subject of study since long. The first contribution in this regard came from Dalton, who postulated that matter is made of atoms, which are indivisible. The word atom comes from the Greek word **atomos** meaning 'not cut'. J.J. Thomson proposed a structure for the atom, which was modified by Rutherford and later by Niels Bohr. In this unit, we shall discuss these *models of atoms* in some detail. We shall also study some fundamental information about the *atomic nuclei*.

The phenomenon of ***'Radioactivity'*** has probably played the most significant role in the development of both, the Atomic and Nuclear Physics. We shall study important aspects of this phenomenon. Further, ***nuclear reactions*** provide useful information regarding interaction of nuclei. This led us to the phenomena of ***nuclear fission*** and ***nuclear fusion*** whereby the vast reservoirs of ***nuclear energy*** were discovered.

DO YOU KNOW ?

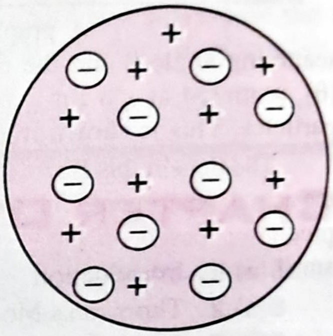
Just as dots of light of only three colours combine to form almost every conceivable colour on T.V. screen, only about 100 distinct kinds of atoms combine to form all the materials in this universe.

We are all made of the same kinds of atoms. These atoms cycle from person to person as we breathe and as our perspiration is vaporized. Atoms are thus recyclable. Our brain may be made of atoms that were once part of Albert Einstein or of Charlie Chaplin, and so on.

8(a).2. THOMSON'S MODEL OF ATOM

According to Thomson's model, every atom consists of a positively charged sphere of radius of the order of 10^{-10} m in which entire mass and positive charge of the atom are uniformly distributed. Inside this sphere, the electrons are embedded like seeds in a watermelon or like plums in a pudding. The number of electrons is such that their negative charge is equal to the positive charge of the atom, Fig. 8(a).1. Thus the atom is electrically neutral.

FIGURE 8(a).1



Limitations of Thomson atom model were the following :

1. *It could not explain the origin of spectral series of hydrogen and other atoms, observed experimentally.*

2. *It could not explain large angle scattering of α particles from thin metal foils, as observed by Rutherford.*

DO YOU KNOW ?

Hydrogen, the smallest atom has a diameter of about 1 Å. A small dot (·) as shown has a diameter of about 1 mm = 10^{-7} Å. Therefore, number of hydrogen atoms that would form a line across this dot = $1/10^{-7} = 10^{7}$ = 10 million or one crore. Imagine how small is an atom !

8(a).3. RUTHERFORD'S α-RAY SCATTERING EXPERIMENT : DISCOVERY OF ATOMIC NUCLEUS

It is an important experiment, which led Rutherford to the discovery of atomic nucleus.

An alpha particle is helium nucleus containing 2 protons and 2 neutrons. Therefore, an alpha particle has 4 units of mass and two units of positive charge. Many radioactive elements emit alpha particles. ***The experimental set up used by Rutherford and his collaborators, Geiger and Marsden is shown in Figs. 8(a).2 and 8(a).3.***

FIGURE 8(a).2

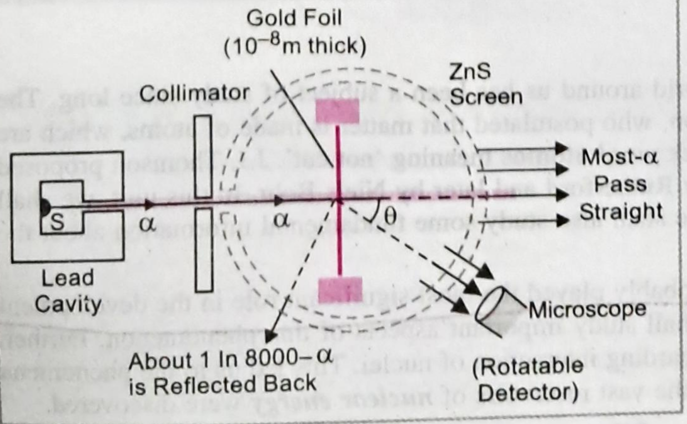
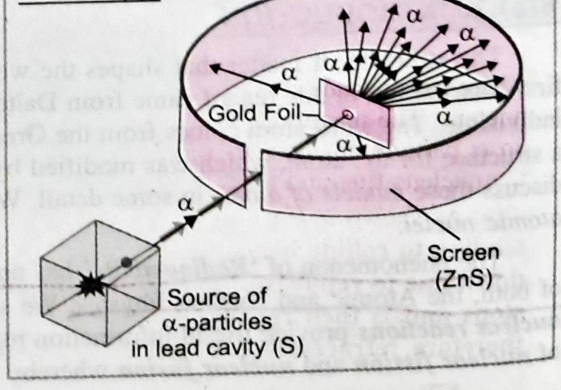


FIGURE 8(a).3

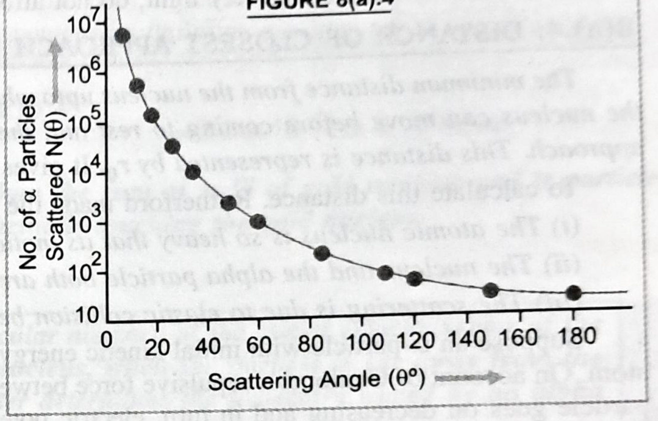


S is a speck of a radioactive source* contained in a lead cavity. The alpha particles emitted by the source are collimated into a narrow beam with the help of a lead slit (collimator). The collimated beam is allowed to fall on a thin gold foil of thickness of the order of $2{\cdot}1 \times 10^{-7}$ m. The α-particles scattered in different directions are observed through a rotatable detector consisting of a circular zinc sulphide screen and a microscope. The alpha particles produce bright flashes or scintillations on the ZnS screen. These are observed in the microscope and counted at different angles from the direction of incidence of the beam. The angle θ of deviation of an alpha particle from its original direction is called its **scattering angle θ.**

IMPORTANT NOTE

This experiment was conducted inside an evacuated chamber to avoid scattering of α–particles due to atoms of the air.

FIGURE 8(a).4



Observations. A graph is plotted between the scattering angle θ and the number of α-particles $N(\theta)$, scattered at $\angle\theta$ for a very large number of α-particles. This is shown in Fig. 8(a).4.

The dots in this figure represent the data points of the actual experiment. The solid curve is the theoretical prediction based on the assumption that atom has a small, dense, positively charged nucleus. We find that

(i) most of the alpha particles pass straight through the gold foil. It means they do not suffer any collision with gold atoms.

(ii) only about 0·14% of incident α-particles scatter by more than 1°.

(iii) About one α particle in every 8000 α particles deflects by more than 90°.

Explanation. The scattering of α-particles is due to Coulombian interaction of α-particles with positive charges and electrons in every atom of the gold foil. As most of the alpha particles pass straight through the foil without any deflection, it means they did not suffer any collision. From this, Rutherford concluded that **most of the space in an atom is empty.**

An α particle is over 7000 times more massive than an electron, and in this experiment, α particle is travelling at a high speed, therefore, very strong forces alone could have deflected them through large angles. This led Rutherford to postulate that the ***entire positive charge of the atom must be concentrated in a tiny central core of the atom. This tiny central core of each atom was called atomic nucleus.***

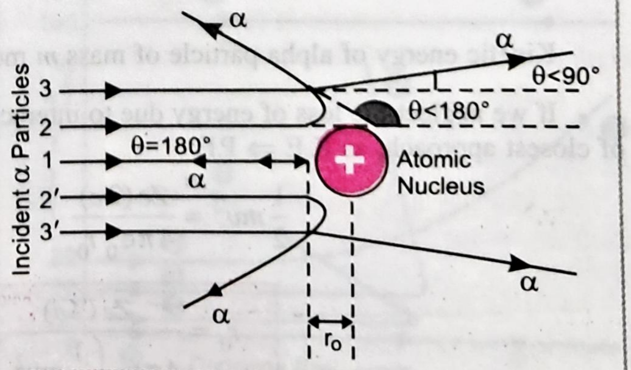
As the gold foil is very thin, it can be assumed that α particles will suffer not more than one scattering during their passage through it. An α particle carries two units of positive charge and has mass of a helium atom. Charge on gold nucleus = Ze, where atomic number of gold, $Z = 79$. As gold nucleus is about 50 times heavier than an α-particle, we assume that it would remain stationary in the scattering process. Therefore, the trajectory of α particle can be computed using Newton's second law of motion and coulomb force of repulsion between α particle and gold nucleus, *i.e.*,

$$F = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(2e)}{r^2}$$

where r is distance of α particle from the centre of the nucleus. The magnitude and direction of the force on an α particle changes continuously as it approaches the nucleus first and then recedes away from it.

As shown in Fig. 8(a).5, an **alpha particle (1), tending to collide head on with the nucleus, slows down due to repulsive force of the nucleus, finally stops and is then repelled back. This α particle, therefore, retraces its path, scattering through 180°.**

FIGURE 8(a).5



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*The source used in this experiment was $_{83}\mathrm{Bi}^{214}$

Alpha particles 2, 2′ tending to hit the nucleus at its periphery, experience strong repulsive forces and get scattered through large angles ($\theta > 90^\circ$)

The alpha particles 3, 3′ which pass at a distance from the nucleus, experience small repulsive forces and get scattered through small angles. The α particles which pass at still larger distances from the nucleus go almost undeviated.

We can show that number of α particles scattered per unit area, $N(\theta)$ at scattering angle θ varies inversely as $\sin^4(\theta/2)$

$$\boxed{N(\theta) \propto \frac{1}{\sin^4(\theta/2)}}$$

Note that electrons being very light, do not affect the motion of alpha particles.

8(a).4. DISTANCE OF CLOSEST APPROACH (SIZE OF NUCLEUS)

The minimum distance from the nucleus upto which an energetic α particle travelling directly towards the nucleus can move before coming to rest and then retracing its path is known as distance of closest approach. This distance is represented by r_0. It gives us the order of the size of the nucleus.

To calculate this distance, Rutherford made the following **assumptions.**

(i) The atomic nucleus is so heavy that its motion during the impact is disregarded.

(ii) The nucleus and the alpha particle both are taken as point charges having no dimensions.

(iii) The scattering is due to elastic collision between nucleus and α-particle.

Suppose an α-particle with initial kinetic energy E is directed towards the centre of the nucleus of an atom. On account of Coulomb's repulsive force between nucleus and alpha particle, kinetic energy of alpha particle goes on decreasing and in turn, electric potential energy of the particle goes on increasing. At a certain distance r_0 from the nucleus, K.E of α particle reduces to zero. The particle stops and it cannot go closer to the nucleus. It is repelled by the nucleus and therefore, it retraces its path, turning through 180°. Therefore, the distance r_0 is known as the **distance of closest approach.** *At this distance, the entire K.E. of α particle is converted into electric potential energy.*

Now, charge on α particle, $q_1 = +2e$; Charge on nucleus, $q_2 = +Ze$,

where Z is the atomic number of material of the foil and $+e$ is charge on a proton.

Electric potential at distance r_0 due to the nucleus $= \dfrac{Ze}{4\pi\epsilon_0 r_0}$ ...(1)

where $\dfrac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

$\therefore$ Potential energy of alpha particle at this distance (r_0) from the nucleus = potential × charge

$$= \frac{Ze}{4\pi\epsilon_0 r_0} \times (2e) = \frac{Ze(2e)}{4\pi\epsilon_0 r_0} \quad \text{...(2)}$$

Kinetic energy of alpha particle of mass m moving with velocity v is $E = \dfrac{1}{2}mv^2$ ...(3)

If we neglect the loss of energy due to interaction of α particle with the electrons, then at the distance of closest approach, as K.E $\Rightarrow$ P.E

$$\therefore \quad \frac{1}{2}mv^2 = \frac{Ze(2e)}{4\pi\epsilon_0 r_0}$$

$$\boxed{r_0 = \frac{Ze(2e)}{4\pi\epsilon_0\left(\frac{1}{2}mv^2\right)}} \quad \text{...(4)}$$

Obviously, the radius of the nucleus must be smaller than the calculated value of r_0, as an alpha particle cannot touch the periphery of the nucleus on account of strong repulsion.

Sample Problem **In the original experiment, Geiger and Marsden calculated the distance of closest approach to the gold nucleus (Z = 79) – of a 7·7 MeV α particle before it comes momentarily to rest and reverses its direction. What is its value ?**

NCERT Solved Example

Sol. *In the original experiment,* K.E. of α particle,

$$E = \frac{1}{2}mv^2 = 7{\cdot}7\text{ MeV} = 7{\cdot}7 \times 1{\cdot}6 \times 10^{-13}\text{ joule}, \quad i.e., \quad E = 1{\cdot}2 \times 10^{-12}\text{ joule}$$

$Z = 79$ for gold and $e = 1{\cdot}6 \times 10^{-19}$ coulomb.

From (4),
$$r_0 = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(2e)}{\frac{1}{2}mv^2} \;; \quad r_0 = \frac{9\times10^9\times79\times2(1{\cdot}6\times10^{-19})^2}{1{\cdot}2\times10^{-12}}$$

$$r_0 = \frac{9\times79\times2\times1{\cdot}6\times1{\cdot}6\times10^{-29}}{1{\cdot}2\times10^{-12}} \;; \quad r_0 = 3{\cdot}0 \times 10^{-14}\text{ m} = \mathbf{30\ fermi}$$

Note that this value is considerably larger than the sum of radii of gold nucleus and α particle. Thus α particle reverses its motion without ever actually touching the gold nucleus.

8(a).5. IMPACT PARAMETER

***Impact parameter** is defined as the perpendicular distance of the initial velocity vector of the alpha particle from the central line of the nucleus, when the particle is far away from the nucleus of the atom. Infact, impact parameter determines the trajectory traced by an alpha particle in passing through the gold foil.*

The angle between the direction of approach of the alpha particle and the direction of scattered alpha particle is called **scattering angle.** *It is denoted by* θ.

FIGURE 8(a).6

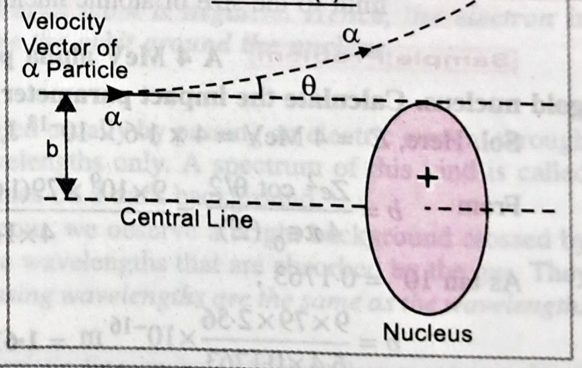


Fig. 8(a).6 shows the path or trajectory of an alpha particle in the coulomb field of a heavy nucleus. The impact parameter b and scattering angle θ are also shown in the diagram.

For large impact parameters, force experienced by the alpha particle is weak, because F varies as 1/(distance)^2. Therefore, when impact parameter is large, an alpha particle will deviate through a much smaller angle. However, when impact parameter is small, force experienced is large and hence the alpha particle will scatter through a large angle. For the case of head on collision, impact parameter b tends to zero. The alpha particle will rebound like a ball thrown against a wall, scattering through 180°.

FIGURE 8(a).7

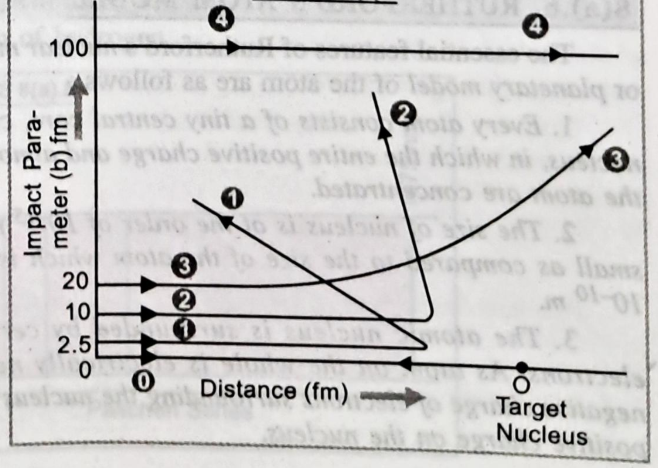


Fig. 8(a).7 shows theoretically calculated paths of alpha particles moving with a speed of $1{\cdot}63 \times 10^7$ m/s in the coulomb field of a gold nucleus. The gold nucleus is supposed to be at the origin O of the coordinate system. The values of impact parameters chosen for alpha particles 1, 2, 3, 4 respectively are 2·5 *fm*, 10 *fm*, 20 *fm* and 100 *fm*. For $b = 0$, α particle 0 retraces its path, turning through 180°, as shown in Fig. 8(a).7.

Rutherford calculated *analytically*, the relation between the impact parameter b and scattering angle θ, which is given by

$$\boxed{b = \frac{1}{4\pi\epsilon_0} \frac{Ze^2 \cot\theta/2}{E}} \quad ...(5)$$

where $E = \frac{1}{2}mv^2$ is kinetic energy of alpha particle, when it is far away from the atom.

DO YOU KNOW ?

The relation between the impact parameter b and the scattering angle θ depends on the nature of the force law between the target scatterer and the alpha particle.

Discussion.

(*i*) For large values of impact parameter b, $\cot\theta/2$ is large. Therefore, scattering angle θ is small. It means, when impact parameter is large, scattering angle is small, *i.e.*, α particles passing far away from the nucleus undergo small deflections.

(*ii*) When impact parameter b is small, $\cot\theta/2$ is small. Therefore, scattering angle θ is large. It means alpha particles passing close to the nucleus suffer large deflections.

(*iii*) When $b = 0$, $\cot\theta/2 = 0$, $\frac{\theta}{2} = 90°$, $\theta = 180°$. Therefore, an alpha particle travelling directly towards the centre of nucleus will retrace its path.

(*iv*) If KE (E) of alpha particle is large, impact parameter b shall be small for the same deflection.

A given beam of alpha particles has some distribution of impact parameters (b). Therefore, the beam is scattered in various directions with different probabilities.

RETAIN IN MEMORY Alpha particle trajectories suggest that the size of target nucleus is smaller compared with the distance of closest approach from a nucleus (r_0) or even the impact parameter (b). Therefore, Rutherford scattering experiment is a powerful way to determine upper limit to the size of atomic nucleus.

Sample Problem **A 4 MeV alpha particle is scattered through 20°, when it approaches a gold nucleus. Calculate the impact parameter if Z for gold is 79.**

Sol. Here, $E = 4\text{ MeV} = 4 \times 1{\cdot}6 \times 10^{-13}$ J, $\theta = 20°$, $Z = 79$, $b = ?$

From $$b = \frac{Ze^2 \cot\theta/2}{4\pi\epsilon_0\,(E)} = \frac{9\times10^9 \times 79 (1{\cdot}6\times10^{-19})^2 \cot 10°}{4\times1{\cdot}6\times10^{-13}}$$

As $\tan 10° = 0{\cdot}1763$;

$$b = \frac{9\times79\times2{\cdot}56}{6{\cdot}4\times0{\cdot}1763}\times10^{-16}\text{ m} = \mathbf{1{\cdot}61 \times 10^{-13}\text{ m}}$$

8(a).6. RUTHERFORD'S ATOM MODEL

The essential features of Rutherford's *nuclear model* of the atom or *planetary model* of the atom are as follows :

1. *Every atom consists of a tiny central core, called the atomic nucleus, in which the entire positive charge and almost entire mass of the atom are concentrated.*

2. *The size of nucleus is of the order of 10^{-15} m, which is very small as compared to the size of the atom which is of the order of 10^{-10} m.*

3. *The atomic nucleus is surrounded by certain number of electrons. As atom on the whole is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.*

DO YOU KNOW ?

Rutherford's Atom Model is analogous to the solar system. The nucleus of an atom plays the role of the sun. The electrons revolving around the nucleus correspond to the planets revolving in their orbits around the sun. The centripetal force needed by planets is provided by gravitational pull of the sun on the planets. Similarly, centripetal force required by revolving electrons is provided by Coulomb's force of attraction between the electrons and the nucleus.

4. *These electrons revolve around the nucleus in various circular orbits as do the planets around the sun. The centripetal force required by electron for revolution is provided by the electrostatic force of attraction between the electrons and the nucleus.*

8(a).7. ELECTRON ORBITS

If F_c = centripetal force required to keep a revolving electron in orbit,

F_e = electrostatic force of attraction between the revolving electron and the nucleus, then for a dynamically stable orbit in a hydrogen atom, $F_c = F_e$

$$m\frac{v^2}{r} = \frac{(e)(e)}{4\pi\epsilon_0 r^2} \qquad ...(6)$$

$$r = \frac{e^2}{4\pi\epsilon_0 mv^2} \quad \text{or} \quad v^2 = \frac{e^2}{4\pi\epsilon_0 mr} \qquad ...(7)$$

K.E. of electron in the orbit, $K = \frac{1}{2}mv^2$ using (6), $\boxed{K = \frac{e^2}{8\pi\epsilon_0 r}}$

Potential energy of electron in orbit, $\boxed{U = \frac{(e)(-e)}{4\pi\epsilon_0 r} = \frac{-e^2}{4\pi\epsilon_0 r}}$

Negative sign indicates that revolving electron is bound to the positive nucleus.

∴ Total energy of electron in hydrogen atom, $E = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$

$$\boxed{E = -\frac{e^2}{8\pi\epsilon_0 r}} \qquad ...(8)$$

Therefore, total energy of electron in orbit of hydrogen atom is negative. Hence, the electron is bound to the nucleus, i.e., the electron is not free to leave the orbit around the nucleus.

8(a).8. ATOMIC SPECTRA

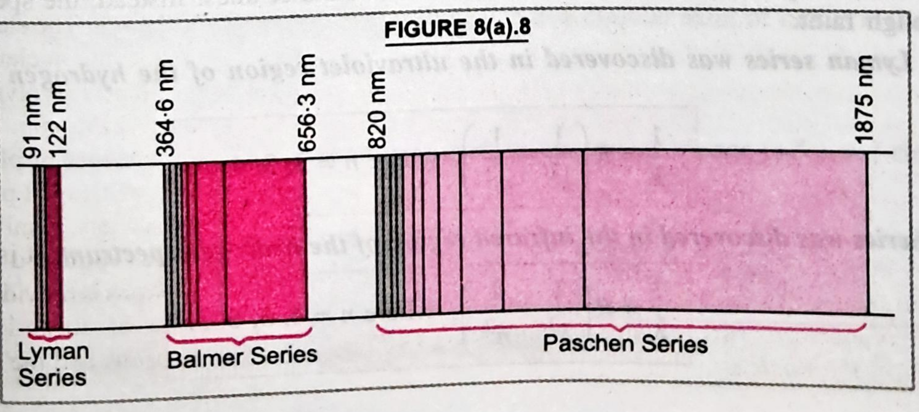
When an atomic gas or vapour at low pressure is excited usually by passing an electric current through it, the gas/vapour emits radiations of certain specific wavelengths only. A spectrum of this kind is called **Line emission spectrum** and it consists of a few bright lines on a dark background.

When white light is passed through the same gas/vapour, we observe a bright background crossed by a few dark lines signifying the **missing wavelengths** or the wavelengths that are absorbed by the gas. They form the **Line Absorption Spectrum.** It was found that *missing wavelengths are the same as the wavelengths present in the emission spectrum of the gas/vapour.*

The fact that every gas/vapour has its own characteristic line emission/absorption spectrum shows that the line spectra serve as finger prints for identification of the gas.

Fig. 8(a).8 shows emission lines in the spectrum of hydrogen.

FIGURE 8(a).8

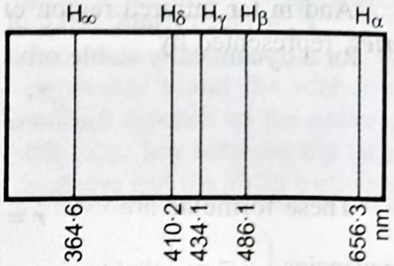


8(a).9. SPECTRAL SERIES OF HYDROGEN

A close look at Fig. 8(a).8 shows that the spectral lines are in groups. Further, the spacing between lines within certain sets of the hydrogen spectrum decreases in a regular way. Each of these sets is called a **spectral series.**

Balmer was the first to observe one such spectral series in the visible region of the hydrogen spectrum. It is called Balmer series and is shown in Fig. 8(a).9. The spectral line with largest wavelength, 656·3 nm in the red region is called H_α line, the next line with $\lambda = 486{\cdot}1$ nm in the blue, green region is called H_β line, the next line with $\lambda = 434{\cdot}1$ nm in violet region is called H_γ line and so on, Table 8(a).1. The spacing of successive lines and their intensity goes on decreasing.

FIGURE 8(a).9



Balmer Series

TABLE 8(a).1. Spectral lines of Balmer Series

S. No.	NAME OF LINE	COLOUR OF LINE	WAVELENGTH
1.	H_α	Red	656·3 nm
2.	H_β	Green	486·1 nm
3.	H_γ	Blue	434·1 nm
4.	H_δ	Violet	410·2 nm

Balmer found an **empirical formula** to account for these wavelengths :

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \text{ where } n = 3, 4, 5, \ldots\ .} \qquad \ldots(8)$$

where R is a constant $= 1{\cdot}097 \times 10^7\ \text{m}^{-1}$. It is called ***Rydberg constant*** and n is an integer having values 3, 4, 5, ...etc.

For $n = 3$, eqn. (8) gives

$$\frac{1}{\lambda} = 1{\cdot}097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1{\cdot}5236 \times 10^6\ \text{m}^{-1}$$

$$\lambda = 656{\cdot}3 \text{ nm}$$

which is the wavelength of H_α line.

Similarly, for $n = 4$, we get $\lambda = 486{\cdot}1$ nm and so on. For $n = \infty$; we get $\lambda = 364{\cdot}6$ nm, which is the ***limit of the Balmer series.*** Beyond this limit, there are no further distinct lines. Instead, the spectrum becomes continuous, though faint.

Later on, ***Lyman series was discovered in the ultraviolet region of the hydrogen spectrum.*** It is represented by

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \text{ where } n = 2, 3, 4, \ldots\ .}$$

Paschen Series was discovered in the infrared region of the hydrogen spectrum. It is represented by

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \text{ where } n = 4, 5, 6, \ldots\ .}$$

Another spectrum, called ***Brackett series was discovered in the infrared region of hydrogen spectrum.*** It is represented by

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \text{ where } n = 5, 6, 7, \ldots .$$

And in far infrared region of hydrogen spectrum, there was yet another spectral series called ***Pfund series***, represented by

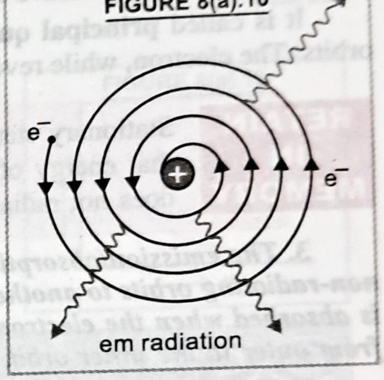
$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \text{ where } n = 6, 7, 8, \ldots$$

These formulae are useful as they give the wavelengths λ and hence frequencies $\left(\nu = \frac{c}{\lambda}\right)$ that hydrogen atoms radiate or absorb. **However, there is no reasoning why certain specific frequencies/wavelengths alone are observed in the hydrogen spectrum.**

8(a).10. LIMITATIONS OF RUTHERFORD ATOM MODEL

In Rutherford atom model, an electron is revolving around the nucleus and is constantly experiencing a centripetal force. Therefore, the electron has an accelerated motion. According to classical electromagnetic theory, the electron must radiate energy in the form of electromagnetic waves. ***As the revolving electron loses energy continuously, it must spiral inwards and eventully fall into the nucleus*****, as shown in Fig. 8(a).10. Thus, Rutherford atom model does not explain the stability of the atom.**

FIGURE 8(a).10



Further, according to classical electromagnetic theory, frequency of *em* waves emitted by revolving electron = frequency of revolution of electron. As the revolving electrons spiral inwards, their angular velocities and hence their frequencies of revolution would change continuously. Therefore, frequency of *em* waves emitted must change continuously. Therefore, ***atoms should emit continuous spectrum, but what we observe is only a line spectrum.***

8(a).11. BOHR MODEL OF HYDROGEN ATOM

In view of the limitations of Rutherford atom model, Niels Bohr came to the conclusion that classical mechanics and electromagnetism could not be applied to the processes on the atomic scale. Bohr cleverly combined classical ideas and early quantum concepts (given by Planck) to give what is known as Bohr Model of hydrogen atom. Following are the ***three basic postulates of this model :***

1. ***Every atom consists of a central core called nucleus, in which entire positive charge and almost entire mass of the atom are concentrated. A suitable number of electrons (having as much negative charge as the positive charge on the nucleus) revolve around the nucleus in circular orbits. The centripetal force required for revolution is provided by the electrostatic force of attraction between the electron and the nucleus.***

If m is the mass of electron moving with a velocity v in a circular orbit of radius r, then the necessary centripetal force is $F = \frac{mv^2}{r}$

Also, the electrostatic force of attraction between the nucleus of charge $(+Ze)$ and electron of charge $(-e)$ is

$$F = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(e)}{r^2} = \frac{KZe^2}{r^2} \quad \text{where } K = \frac{1}{4\pi\epsilon_0}$$

---

*For hydrogen atom, $Z = 1$, as it contains only one proton.

$$\therefore \quad \boxed{\frac{mv^2}{r} = \frac{KZ^{*}e^2}{r^2}} \qquad \text{...(9)}$$

2. According to Bohr, electron can revolve only in certain discrete non radiating orbits, called stationary orbits, for which total angular momentum of the revolving electron is an integral multiple of $h/2\pi$, where h is Planck's constant.

Thus the angular momentum of the orbiting electron is quantised.

As angular momentum of electron = mvr, $\therefore$ for any permitted (stationary) orbit

$$\boxed{mvr = \frac{nh}{2\pi}} \qquad \text{...(10)}$$

where n is any positive integer, 1, 2, 3.....

It is called **principal quantum number.** The quantum condition (10) limits the number of allowed orbits. The electron, while revolving in such orbits, shall not lose energy *i.e.,* its energy would stay constant.

RETAIN IN MEMORY

Stationary state does not mean that electron is stationary in that state. Rather, it means that energy of electron revolving in stationary orbit remains constant, *i.e.,* the electron does not radiate/emit/lose any energy while moving in a stationary orbit.

3. The emission/absorption of energy occurs only when an electron jumps from one of its specified non-radiating orbits to another. The difference in the total energy of electron in the two permitted orbits is absorbed when the electron jumps from an inner to an outer orbit, and emitted when electron jumps from outer to the inner orbit.

If E_1 is total energy of electron in an inner stationary orbit and E_2 is its total energy in an outer stationary orbit, then *frequency* ν *of radiation emitted on jumping from outer to inner orbit is given by*

$$\boxed{h\nu = E_2 - E_1} \qquad \text{...(11)}$$

On the basis of these three postulates, we calculate the following :

(a) Radii of Bohr's stationary orbits

From (10), $\quad mvr = \dfrac{nh}{2\pi}, \quad$ or $\quad v = \dfrac{nh}{2\pi mr}$

Put in (9), $\quad \dfrac{m}{r}\dfrac{n^2h^2}{4\pi^2 m^2 r^2} = \dfrac{KZe^2}{r^2} \quad$ or $\quad r = \dfrac{n^2h^2}{4\pi^2 m KZe^2} = \dfrac{n^2}{Z}\left(\dfrac{h^2}{4\pi^2 m Ke^2}\right) \qquad$ ...(12)

This shows that $\boxed{r \propto n^2 \text{ and } r \propto \frac{1}{Z}}$

i.e., radii of stationary orbits are in the ratio $1^2 : 2^2 : 3^2$: and so on i.e. 1 : 4 : 9. Clearly, the stationary orbits are not equally spaced.

The variation of radius (r) of stationary orbit of hydrogen atom with the principal quantum number is shown in Fig. 8(a).11.

Putting $h = 6{\cdot}6 \times 10^{-34}$ joule-sec., $m = 9{\cdot}1 \times 10^{-31}$ kg,

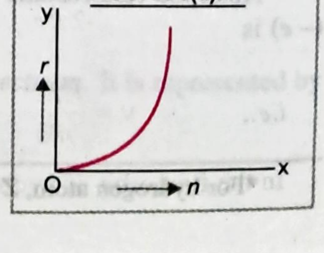
$K = 9 \times 10^9$ Nm^2 C^{-2}, $e = 1{\cdot}6 \times 10^{-19}$ C

we get from eqn. (12) $\qquad r = n^2 \times 5{\cdot}29 \times 10^{-11}$ m

For example, size of 1st orbit (n =1) of hydrogen atom is

$\therefore \qquad r = 1 \times 5{\cdot}29 \times 10^{-11}$ m $= 0{\cdot}529$ Å $= 5{\cdot}29 \times 10^{-11}$ m

FIGURE 8(a).11



Eqn. (12) can be rewritten as

$$\boxed{r_n = (5{\cdot}29\times10^{-11}\ \text{m})\frac{n^2}{Z}} \quad ...(13)$$

(b) Velocity of electron in Bohr's stationary orbit

From (9), $r = \frac{KZe^2}{mv^2}$ From (10), $r = \frac{nh}{2\pi mv}$

$$\therefore \quad \frac{KZe^2}{mv^2} = \frac{nh}{2\pi mv} \quad \text{or} \quad v = \frac{2\pi KZe^2}{nh} \quad ...(14)$$

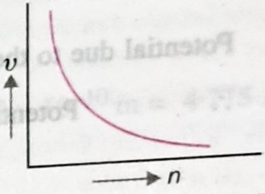
For hydrogen atom, Z = 1

$$\therefore \quad \boxed{v = \frac{2\pi Ke^2}{nh}} \quad ...(15)$$

Calculations show that *in the first orbit (n = 1) of hydrogen atom, orbital velocity of electron is 2·2 × 10⁶ m/s which is roughly $\frac{1}{137}$ of the velocity of light in vacuum.*

Further, the orbital velocity of electron in outer orbits is smaller as compared to its value in the inner orbits. ($\because v \propto 1/n$)

FIGURE 8(a).12



The variation of speed (v) of an electron in a stationary orbit of hydrogen atom with the principal quantum number (n) is shown in Fig. 8(a).12.

Sample Problem Find the wavelength of electron orbiting in the first excited state of hydrogen atom. (CBSE 2017)

Sol. Here, $\lambda = ?$ $n = 2$ for first excited state

$$v = \frac{2\pi Ke^2}{nh}$$

$$\lambda = \frac{h}{mv} = \frac{h,\ nh}{2\pi m Ke^2} = \frac{nh^2}{2\pi m Ke^2}$$

$$\lambda = \frac{2(6{\cdot}6\times10^{-34})^2}{2\times3{\cdot}14\times9\times10^{-31}(9\times10^9)(1{\cdot}6\times10^{-19})^2}$$

$$\lambda = \frac{2\times6{\cdot}6\times6{\cdot}6\times10^{-8}\ \text{m}}{6{\cdot}28\times81\times1{\cdot}6\times1{\cdot}6} = \mathbf{6{\cdot}69\times10^{-10}\ m}$$

(c) Frequency of electron in Bohr's stationary orbit

It is the number of revolutions completed per second by the electron in a stationary orbit, around the nucleus. It is represented by ν

From $v = r\,\omega = r\,(2\,\pi\,\nu)$

$$\nu = \frac{v}{2\pi r} = \frac{2\pi KZe^2}{nh\cdot 2\pi r} = \frac{KZe^2}{nhr}, \quad \text{using (15),}$$

i.e., $$\boxed{\nu = \frac{KZe^2}{nhr}} \quad ...(16)$$

In the first orbit of hydrogen atom, $n = 1$, $r = 0{\cdot}53 \times 10^{-10}$ m

DO YOU KNOW ?

We can rewrite eqn. (15) as

$$v = \frac{2\pi Ke^2}{nh} = \left(\frac{2\pi Ke^2}{ch}\right)\frac{c}{n}$$

$$\boxed{v = \alpha\frac{c}{n}}$$

where $\alpha = \frac{2\pi Ke^2}{ch}$ = constant, called **fine structure constant.**

$$\alpha = \frac{2\times3{\cdot}14\times9\times10^9\ (1{\cdot}6\times10^{-19})^2}{(3\times10^8)(6{\cdot}66\times10^{-34})}$$

$$\boxed{\alpha = \frac{1}{137}}$$

∴ In the first stationary orbit of hydrogen atom, orbital velocity of electron,

$$v = \alpha\frac{c}{n} = \frac{1}{137}\times\frac{3\times10^8}{1}$$

$$= \mathbf{2{\cdot}2\times10^6\ m/s}$$

using $K = 9 \times 10^9$ Nm^2 C^{-2}, $Z = 1$, $h = 6{\cdot}6 \times 10^{-34}$ Js, $e = 1{\cdot}6 \times 10^{-19}$ C, we get

$$\nu = \frac{9\times10^9 \times 1 \times (1{\cdot}6\times10^{-19})^2}{(6{\cdot}6\times10^{-34})\times(0{\cdot}53\times10^{-10})} = 6{\cdot}57 \times 10^{15} \text{ rps}$$

As $r \propto n^2$, therefore from (16), $\boxed{\nu \propto \frac{1}{n^3}}$

The frequency of electron in subsequent stationary orbits is smaller.

(d) Total energy of electron in Bohr's stationary orbit

The energy of elecrtron revolving in a stationary orbit is of two types : ***Kinetic energy which is due to velocity and potential energy which is due to position of electron.***

From the first postulate of Bohr's atom model, $\frac{mv^2}{r} = \frac{KZe^2}{r^2}$

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}\frac{KZe^2}{r} \quad \textit{i.e.,} \quad \text{K.E of electron} = \frac{1}{2}mv^2 = \frac{KZe^2}{2r} \quad \text{...(17)}$$

Potential due to the nucleus, at any point in the orbit in which electron is revolving $= \frac{KZe}{r}$

$\therefore$ **Potential energy of electron = potential × charge**

$$= \frac{KZe(-e)}{r} = \frac{-KZe^2}{r} \quad \text{...(18)}$$

Total energy of electron in the orbit, $E = K.E. + P.E. = \frac{1}{2}\frac{KZe^2}{r} - \frac{KZe^2}{r} = -\frac{KZe^2}{2r}$

Putting $r = \frac{n^2h^2}{4\pi^2 mKZe^2}$ **from (12), we get ,**

$$\boxed{E = -\left(\frac{2\pi^2 m K^2 e^4}{h^2}\right)\frac{Z^2}{n^2}} \quad \text{...(19)}$$

Substituting the standard values, we get

$$E = -(21{\cdot}76\times10^{-19})\frac{Z^2}{n^2}\text{J} = -\frac{21{\cdot}76\times10^{-19}Z^2}{n^2\times1{\cdot}6\times10^{-19}}\text{eV} = -(13{\cdot}6)\frac{Z^2}{n^2}\text{eV} \quad \text{...(20)}$$

For hydrogen atom, $Z = 1$

$$\therefore \quad \boxed{E = -\frac{13{\cdot}6}{n^2}\text{eV}} \quad \text{...(21)}$$

Eqn. (21) shows that *total energy of electron in a stationary orbit is negative, which means the electron is bound to the nucleus and is not free to leave it.*

Further, as *n* has discrete values (1, 2, 3, ...), therefore energy of an electron in hydrogen atom is quantised. This is called Bohr's energy quantisation.

RETAIN IN MEMORY

In ground state of hydrogen atom (Z = 1, *n* = 1), From (17) and (18),

$$K.E. = \frac{1}{2}\frac{Ke^2}{r} \quad \text{and} \quad P.E. = -\frac{Ke^2}{r} = -2\,K.E.$$

$$\text{Total energy,} \quad E = K.E. + P.E. = \frac{1}{2}\frac{Ke^2}{r} - \frac{Ke^2}{r} = -\frac{1}{2}\frac{Ke^2}{r} = -K.E.$$

Thus in a hydrogen atom, $K.E. = E$; $P.E. = -2E$, $T.E. = -E$

As total energy of electron revolving in any orbit is negative, the revolving electrons are bound to the nucleus.

The energy of the atom is **least** (= largest negative value) when $n = 1$, *i.e.*, when electron is revolving in an orbit closest to the nucleus. This state of lowest energy of the atom is called **ground state.** The energy of this state is, $E_1 = -\frac{13 \cdot 6}{1^2}\,\text{eV} = -13 \cdot 6\,\text{eV}$. Therefore, the minimum energy required to remove the electron from the ground state of hydrogen atom is 13·6 eV. This is called **ionisation energy** of hydrogen atom.

As n increases, value of negative energy decreases, *i.e.*, ***energy is progressively larger in the outer orbits.*** These are called **Excited States.**

RETAIN IN MEMORY Note that stationary orbits of electrons are not equally spaced. Their radii are in the ratio $1^2 : 2^2 : 3^2$ .... In outer stationary orbits, velocity of electrons and their frequency of revolution decrease. Total energy of electron in outer orbits is more than that in inner orbits.

Sample Problem **Calculate the radius of the third Bohr orbit of hydrogen atom and the energy of electron in that orbit.**

Sol. Here, $r = ?, n = 3, E = ?$

$$\text{As} \quad r = \frac{n^2 h^2}{4\pi^2 m k e^2} \quad \therefore \; r = \frac{3^2 (6 \cdot 6 \times 10^{-34})^2 \times 9 \times 10^9}{4 \times \frac{22}{7} \times \frac{22}{7} \times 9 \cdot 1 \times 10^{-31} (1 \cdot 6 \times 10^{-19})^2} = 4 \cdot 775 \times 10^{-10}\,\text{m} = \mathbf{4 \cdot 775\ A^{\circ}}$$

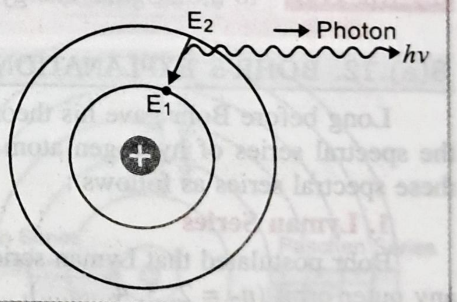
$$E = \frac{-2\pi^2 m k^2 e^4}{n^2 h^2}$$

On putting the standard values, we get $\quad E = -\mathbf{2 \cdot 43 \times 10^{-19}\ J}$

(e) Origin of spectral lines

At room temperature, most of the hydrogen atoms are in ground state. When a hydrogen atom receives energy by processes such as electron collisions or heat, the atom may acquire sufficient energy to raise the electron to higher energy states, *i.e.*, from $n = 1$ to $n = 2, 3, \ldots$. The atom is then said to be in an **excited state.** From these excited states, the electron can fall back to a state of lower energy–emitting thereby a photon of particular energy (= difference in energies of the two states), Fig. 8(a).13.

FIGURE 8(a).13



Suppose

E_1 = total energy of electron in the inner (n_1th) orbit.

E_2 = total energy of electron in the outer (n_2th) orbit.

When an electron jumps from an outer to an inner orbit, the energy of radiation emitted, according to Bohr's third postulate, is $\quad h\nu = E_2 - E_1$

$$\text{Using (19),} \quad h\nu = -\frac{2\pi^2 m K^2 Z^2 e^4}{n_2^2 h^2} + \frac{2\pi^2 m K^2 Z^2 e^4}{n_1^2 h^2}$$

$$\frac{hc}{\lambda} = \frac{2\pi^2 m K^2 Z^2 e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = \frac{2\pi^2 m K^2 e^4 Z^2}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \ldots(22)$$

Now, $\frac{1}{\lambda} = \bar{\nu}$, the *wave number of radiation emitted i.e., number of complete waves in unit length.*

$$\boxed{\frac{2\pi^2 mK^2e^4}{ch^3} = R,}$$ a constant called ***Rydberg constant.***

Putting the values of various terms in the above relation, we get the value of Rydberg constant

$$\boxed{R = 1{\cdot}097 \times 10^7\ \text{m}^{-1}}$$

∴ From (22), $$\boxed{\bar{\nu} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

For hydrogen, $Z = 1$ ∴ $$\boxed{\bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]} \quad ...(23)$$

Eqn. (23) is called ***Rydberg formula for the spectrum of hydrogen atom.*** Clearly, **wavelengths/frequencies/wave numbers of radiations emitted by the excited hydrogen atom are not continuous. They have specific values depending upon the values of n_1 and n_2.**

Sample Problem **In hydrogen atom, wavelength of emitted photon will be minimum in which of the following transitions ?**

***(i)* $n = 2$ to $n = 1$; *(ii)* $n = 6$ to $n = 5$ *(iii)* $n = 3$ to $n = 2$** **(MP PET 2012)**

Sol. Wavelength of emitted photon will be minimum when frequency and energy of photon are maximum. For maximum energy, the energy difference of the two transitions should be maximum. This occurs between $n = 2$ to $n = 1$.

RETAIN IN MEMORY When external energy is supplied to an atom, an electron in any stationary orbit may absorb this energy and go over to higher energy level. After 10^{-8} second, the electron jumps back to the original energy level by emitting the absorbed energy in the form of a photon.

8(a).12. BOHR'S EXPLANATION OF SPECTRAL SERIES OF HYDROGEN ATOM

Long before Bohr gave his theory of hydrogen atom, various scientists had observed experimentally, the spectral series of hydrogen atom as detailed in Art. 8(a).9. Bohr offered a theoretical explanation of these spectral series as follows :

1. Lyman Series

Bohr postulated that Lyman series is obtained when an electron jumps to the first orbit ($n_1 = 1$) from any outer orbit ($n_2 = 2, 3, 4$..........).

Wave numbers of spectral lines of Lyman series were calculated using eqn.(23), *i.e.*,

$$\bar{\nu} = R\left[\frac{1}{1^2} - \frac{1}{k^2}\right], \text{ where } k = 2,3,4.....$$

The *longest wavelength* of the spectral lines of Lyman series is emitted when transition of electron takes place from $n_2 = k = 2$ to $n_1 = 1$

i.e., $$\frac{1}{\lambda_{L_{\max}}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda_{L_{\max}} = \frac{4}{3R} = \frac{4}{3 \times 1{\cdot}097 \times 10^7} = 1{\cdot}216 \times 10^{-7}\ \text{m} = 1216\ \text{Å}$$

The *shortest wavelength* of the spectral series of Lyman series is emitted when transition of electron takes place from $n_2 = k = \infty$ to $n_1 = 1$

i.e., $$\frac{1}{\lambda_{L_{\min}}} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right); \quad \lambda_{L_{\min}} = \frac{1}{R} = \frac{1}{1{\cdot}097 \times 10^7} = 0{\cdot}911 \times 10^{-7} \text{ m} = 912 \text{ Å}$$

These values of $\bar{\nu}$ lie in the ***ultra violet region*** of the spectrum and agree well with the values of $\bar{\nu}$ observed experimentally by Lyman.

2. Balmer Series

According to Bohr, Balmer series is obtained when an electron jumps to the second orbit ($n_1 = 2$) from any outer orbit ($n_2 = 3,4,5...$).

Wave numbers of these spectral lines were calculated using eqn.(23), *i.e.*

$$\bar{\nu} = R\left[\frac{1}{2^2} - \frac{1}{k^2}\right], \text{ where } k = 3, 4, 5......$$

This set of spectral lines lie in the ***visible part*** of the spectrum.

Proceeding as above, we can prove that

$$\lambda_{B_{\max}} = 6563 \text{ Å} \quad \text{and} \quad \lambda_{B_{\min}} = 3646 \text{ Å}$$

3. Paschen Series

According to Bohr, Paschen series is obtained when an electron jumps to the 3rd orbit ($n_1 = 3$) from any outer orbit ($n_2 = 4, 5, 6....$). Bohr calculated the wave numbers of spectral lines of Paschen series from the relation :

$$\bar{\nu} = R\left[\frac{1}{3^2} - \frac{1}{k^2}\right], \text{ where } k = 4, 5, 6.....$$

These values of $\bar{\nu}$ lie in the ***infra red region*** of the spectrum and agree well with values of $\bar{\nu}$ observed experimentally by Paschen.

Proceeding as above, we can prove that

$$\lambda_{P_{\max}} = 18751 \text{ Å}$$

and $$\lambda_{P_{\min}} = 8220 \text{ Å}$$

Bohr also ***predicted*** two new series in the infra red region of the spectrum, which were observed later by Brackett and Pfund.

4. Brackett Series

According to Bohr, Brackett series is obtained when an electron jumps to the 4th orbit ($n_1 = 4$) from any outer orbit ($n_2 = 5, 6, 7...$).

5. Pfund Series

According to Bohr, Pfund series is obtained when an electron jumps to the 5th orbit ($n_1 = 5$) from any outer orbit ($n_2 = 6, 7, 8...$).

The various spectral series of hydrogen atom are represented in Fig. 8(a).14.*

FIGURE 8(a).14

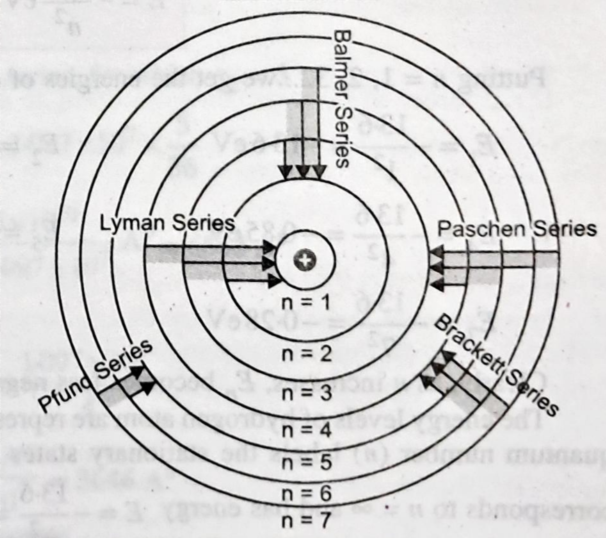


Table 8(a).2 gives the longest wavelengths ($\lambda_{\max}$) and the shortest** wavelengths ($\lambda_{\min}$) of some of the spectral series of hydrogen atom.

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*Circular orbits shown in Fig. 8(*a*).14 are only symbolic. These are not drawn to scale as $r \propto n^2$

**Wavelength emitted will be shortest when $n_2 = \infty$

TABLE 8(a).2. **Wave length limits (λ_{max} and λ_{min}) of some spectral series of Hydrogen**

Spectral Series	Lower state	Upper state	$\lambda_{max.}$	Lower state	Upper state	$\lambda_{min.}$
1. Lyman	1	2	1216 Å	1	∞	912 Å
2. Balmer	2	3	6563 Å	2	∞	3646 Å
3. Paschen	3	4	18751 Å	3	∞	8220 Å

8(a).13. ENERGY LEVEL DIAGRAM

A diagram which represents the total energies of electron in different stationary orbits of an atom is called the energy level diagram of that atom. In this diagram, total energies of electron in various stationary orbits are represented by parallel horizontal lines drawn according to some suitable energy scale.

The vertical line connecting any two states represents the transition of the electron from one to the other of these states. Difference of energies of two states gives the amount of energy emitted/absorbed according as the electron goes from higher to lower energy state or from lower to higher energy level, in an atom.

Total energy of electron in nth orbit of hydrogen atom is

$$E = \frac{-2\pi^2 mK^2 e^4}{n^2h^2} \qquad ...(24)$$

DO YOU KNOW ?

1. Bohr's atomic theory predicted the wavelengths in different series, which are exactly same as found experimentally. That is how authenticity and validity of Bohr's atomic theory is established.

2. There are more spectral series also, *e.g.*, ***Humphrey series*** — which is obtained when electron jumps to 6th orbit from any outer orbit (n = 7, 8, 9....).

On substituting the standard values, we get as shown in Art 8(a).11(d),

$$E = -\frac{13{\cdot}6}{n^2}\,\text{eV} \qquad ...(25)$$

Putting n = 1, 2, 3.....we get the energies of electrons in various stationary orbits as :

$$E_1 = -\frac{13{\cdot}6}{1^2} = -13{\cdot}6\,\text{eV} \qquad E_2 = -\frac{13{\cdot}6}{2^2} = -3{\cdot}4\,\text{eV} \qquad E_3 = -\frac{13{\cdot}6}{3^2} = -1{\cdot}51\,\text{eV}$$

$$E_4 = -\frac{13{\cdot}6}{4^2} = -0{\cdot}85\,\text{eV} \qquad E_5 = -\frac{13{\cdot}6}{5^2} = -0{\cdot}54\,\text{eV} \qquad E_6 = -\frac{13{\cdot}6}{6^2} = -0{\cdot}37\,\text{eV}$$

$$E_7 = -\frac{13{\cdot}6}{7^2} = -0{\cdot}28\,\text{eV}$$

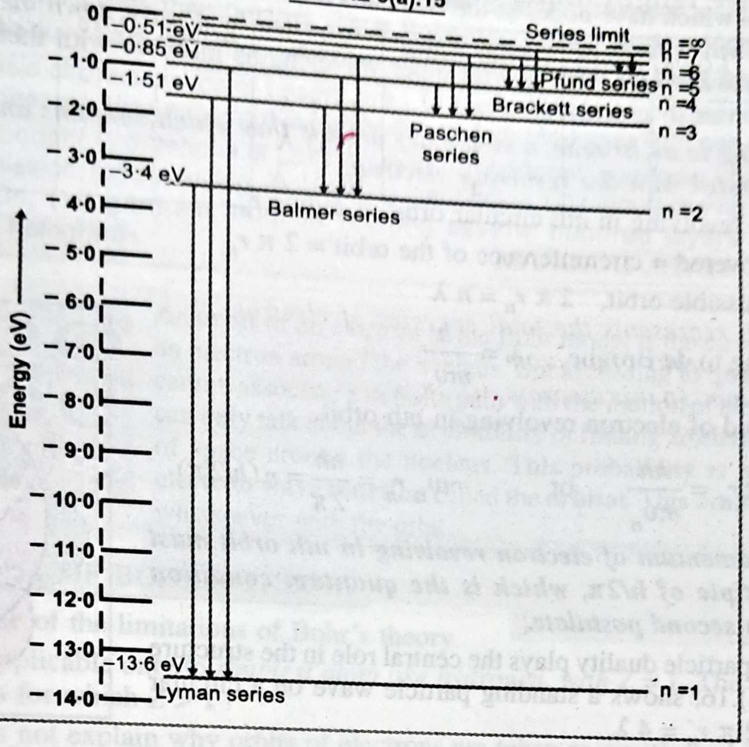
Clearly, as n increases, E_n becomes less negative until at $n = \infty$, $E_n = 0$.

The energy levels of hydrogen atom are represented in energy level diagram, Fig. 8(a).15. The principal quantum number (n) labels the stationary states in ascending order of energy. The highest energy state corresponds to $n = \infty$ and has energy $E = -\frac{13{\cdot}6}{\infty^2} = 0\,\text{eV}$. This is the energy of the atom, when the electron is removed ($r = \infty$) from the nucleus and the electron is at rest. ***As n increases, energies of the excited states come closer and closer together as shown in Fig. 8(a).15.***

Note that an electron can have any total energy above E = 0 eV. In such a situation, the electron is free. And there is a continuum of energy states above E = 0 eV.

The various spectral series of hydrogen atom have also been drawn in Fig. 8(a).15. From this diagram, we can calculate the energy, frequency, wave number etc. of any line of the various spectral series of hydrogen atom.

FIGURE 8(a).15



Sample Problem **Calculate the longest and shortest wavelength in the Balmer series of hydrogen atom. Given Rydberg constant = 1·0987 × 10^7 m^{-1}.**

Sol. The wavelength (λ) of different spectral lines of Balmer series is given by

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{n_2^2}\right]$$

Longest wavelenth is of H_α line or 1st line of the series, for which $n_2 = 3$

$$\therefore \quad \frac{1}{\lambda}=1{\cdot}097\times10^7\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=1{\cdot}097\times10^7\times\frac{5}{36}$$

$$\lambda=\frac{36}{5\times1{\cdot}097\times10^7}\text{ m}=\frac{36\times10^{10}}{5\times1{\cdot}097\times10^7}\text{ A}^\circ=\mathbf{6563\ A^\circ}$$

For shortest wavelength, $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda}=1{\cdot}097\times10^7\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right]=\frac{1{\cdot}097\times10^7}{4}$$

$$\lambda=\frac{4}{1{\cdot}097\times10^7}\text{ m}=\frac{4\times10^{10}\text{ A}^\circ}{1{\cdot}097\times10^7}=\mathbf{3646\ A^\circ}$$

8(a).14. DE BROGLIE'S EXPLANATION OF BOHR'S SECOND POSTULATE OF QUANTIZATION

The second postulate of Bohr atom model says that *angular momentum of electron orbiting around the nucleus is quantized, i.e., $mvr = \frac{nh}{2\pi}$*, where n = 1, 2, 3. Louis de Broglie explained this puzzle in 1923, ten years after Bohr proposed his model. According to de Broglie, the electron in its circular orbit, as proposed by Bohr, must be seen as a particle wave.

We know that when a string fixed at two ends is plucked, a large number of wavelengths are excited. But only those waves which have nodes at the two ends form the standing waves and survive. It means that in a string, *standing waves form when total distance travelled by a wave down the string and back is any integral number of wavelengths.* Waves with other wavelengths interfere with themselves upon reflection and their amplitudes vanish quickly.

Hence, according to de Broglie, a stationary orbit is that which contains an integral number of de Broglie waves associated with the revolving electron.

For an electron revolving in *n*th circular orbit of radius r_n,

total distance covered = circumference of the orbit = $2\pi r_n$

$\therefore$ For the permissible orbit, $2\pi r_n = n\lambda$

According to de Broglie, $\lambda = \dfrac{h}{mv_n}$

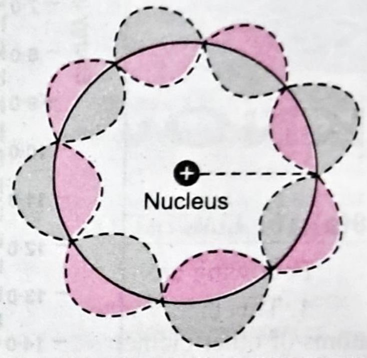
where v_n is speed of electron revolving in *n*th orbit.

$$\therefore \quad 2\pi r_n = \frac{nh}{mv_n} \quad \text{or} \quad mv_n r_n = \frac{nh}{2\pi} = n(h/2\pi)$$

i.e., angular momentum of electron revolving in nth orbit must be an integral multiple of h/2π, which is the quantum condition proposed by Bohr in second postulate.

Thus, the wave particle duality plays the central role in the structure of the atom. Fig. 8(a).16, shows a standing particle wave on a circular orbit for $n = 4$, *i.e.*, $2\pi r_n = 4\lambda$.

FIGURE 8(a).16



Sample Problem **The velocity of electron in inner most orbit of hydrogen atom is $2{\cdot}2 \times 10^6$ m/s. Use de-Broglie hypothesis to calculate radius of innermost orbit.**

Sol. Here, $v = 2{\cdot}2 \times 10^6$ m/s.

de-Broglie wavelength associated with this electron,

$$\lambda = \frac{h}{mv} = \frac{6{\cdot}6\times10^{-34}}{(9\times10^{-31})\,(2{\cdot}2\times10^{6})} = 0{\cdot}333 \times 10^{-9}\ \text{m}$$

If r is radius of the orbit, then according to de-Broglie hypothesis.

$$2\pi r = 1\lambda\,;\ r = \frac{\lambda}{2\pi} = \frac{0{\cdot}333\times10^{-9}}{2\times3{\cdot}14} = 0{\cdot}53 \times 10^{-10}\ \text{m} = \mathbf{0{\cdot}53\ \text{Å}}$$

CURIOSITY QUESTIONS

Q. 1. What is Bohr's correspondence principle ?

Ans. As is known, Bohr's atom model has been replaced by quantum mechanical model. According to this model, electrons in an atom do not move around the nucleus in definite orbits. However, the probability of finding the electron is high near the Bohr orbit radius, and at the same time, the probability of finding the electron between these orbits is not zero.

According to Bohr's correspondence principle, the predictions of quantum theory must correspond to the predictions of classical theory in the regions of sizes where classical theory holds.

For large sizes wherein classical theory holds good, quantum number *n* becomes large. We may therefore rewrite Bohr's correspondence principle as :

$$\lim_{n\to\infty}\ [\text{quantum physics}] = (\text{Classical Physics})$$

For example, quantum condition for emission of radiation is $h\nu = E_i - E_f$

And Maxwell's classical theory says that an electron revolving with orbital frequency f must radiate light waves of frequency f. Calculations show that for quantum number n as large as 10000, the difference in ν and f is less than 0·015%. Thus Bohr's correspondence principle is established.

Q. 2. What are the basic characteristics of Lasers? Mention some of their applications.

Ans. Some of the basic characteristics of Lasers are : (*i*) Laser light is highly monochromatic, (*ii*) Laser light is highly coherent, (*iii*) Laser light is highly directional, (*iv*) Laser light can be sharply focussed. Some of the important applications of Lasers are in : (*i*) performing surgery of many kinds, (*ii*) voice and data transmission, (*iii*) surveying, (*iv*) welding autobodies, (*v*) manufacturing and reading compact discs and DVDs, (*vi*) garment industry (cutting several hundred layers of cloth at a time), (*vii*) generating holograms.

DO YOU KNOW ?

An **orbit** of an electron in the Bohr model is the circular path of motion of an electron around the nucleus. But according to **quantum mechanics,** we cannot associate a definite path with the motion of electrons in an atom. We can only talk about the **probability** of finding an electron in a certain region of space around the nucleus. This probability is inferred from the one **electron wave function** called the **orbital.** This orbital has no resemblence whatsoever with the orbit.

8(a).15. LIMITATIONS OF BOHR'S THEORY

Following are some of the limitations of Bohr's theory.

1. This theory is applicable only to *simplest atom like hydrogen, with* $Z = 1$. The theory fails in case of atoms of other elements for which $Z > 1$.

2. The theory does not explain why orbits of electrons are taken as circular, while elliptical orbits are also possible.

3. Bohr's theory does not explain the *fine structure of spectral lines* even in hydrogen atom.

4. Bohr's theory does not say anything about the *relative intensities of* spectral lines.

5. Bohr's theory does not take into account the *wave properties of* electrons.

6. Bohr's atomic model does not give any indication regarding the arrangement and distribution of electrons in an atom.

7. Bohr's model could not explain the splitting of spectral lines under the effect of magnetic field (Zeeman Effect).

8. Bohr's model could not explain the splitting of spectral lines under the effect of electric field (Stark Effect).

8(a).16. HYDROGEN LIKE ATOMS

As is known, hydrogen atom has a single proton in its nucleus and **a single electron** is revolving around the nucleus in a circular orbit.

An atom consisting of a nucleus of charge + *Ze* (where Z is the atomic number of the atom) but having a single electron revolving around the nucleus is called Hydrogen like atom.

For example :

(*i*) singly ionized Helium atom (${}_2He^4$) has $Z = 2$; and a single electron revolving around the nucleus, as out of two electrons, one has been removed.

(*ii*) doubly ionised lithium atom (${}_3Li^7$) has $Z = 3$, and a single electron revolving around the nucleus, as out of three electrons, two have been removed.

We can show that

(*a*) **Radius** (r'_n) of *n*th orbit of hydrogen like atom $\boxed{r'_n = \frac{r_n}{Z}}$

For example, for singly ionized $_2He^4$ atom, radius of first orbit,

$$(r_0)_{He} = \frac{(r_0)\,H}{Z} = \frac{0{\cdot}53}{2} = \mathbf{0{\cdot}265\ \text{Å}.}$$

(*b*) Speed (v'_n) of electron in *n*th orbit of hydrogen like atom

$$\boxed{v'_n = Z\,(v_n)}$$

For example, speed of electron in 1st orbit of singly ionised He atom

$$v'_0 = Z\,(v_0) = \mathbf{2\ (2 \times 10^6)\ m/s}$$

and speed of electron in 1st orbit of doubly ionised Lithium atom ($Z = 3$) is

$$v'_0 = Z\,(v_0) = \mathbf{3 \times 2 \times 10^6\ m/s}$$

(*c*) Energy of electron in *n*th orbit of hydrogen like atom

$$\boxed{E'_n = \frac{13{\cdot}6\,Z^2}{n^2}\ \text{eV}}$$

For example energy of electron in 1st orbit of singly ionised He atom

$$E'_0 = -\frac{13{\cdot}6 \times 2^2}{1^2}\ \text{eV} = \mathbf{-54{\cdot}4\ eV}$$

and energy of electron in first orbit of doubly ionised Lithium atom

$$E'_0 = \frac{13{\cdot}6 \times 3^2}{1^2}\ \text{eV} = \mathbf{-122{\cdot}4\ eV}$$

Note. Some energy levels of singly ionised He atom are same as that of hydrogen atom. Therefore, spectrum of light emitted by He^+ atom and hydrogen atom may be same.

Sample Problem 1 **Calculate radius of first orbit of singly ionized He atom, when radius of first orbit of hydrogen atom is 0·53 Å.**

Sol. For helium atom, $Z = 2 \quad \therefore\ r_{He} = \frac{r_H}{Z} = \frac{0{\cdot}53\ \text{Å}}{2} = \mathbf{0{\cdot}265\ \text{Å}}$

Sample Problem 2 **The energy of electron in 1st orbit of hydrogen atom is – 13·6 eV. What will be the energy of doubly ionised $_3Li^7$ atom in the first orbit ?**

Sol. For $_3Li^7$, $Z = 3$ and $n = 1$ for first orbit $\quad E' = E\frac{Z^2}{n^2} = -13{\cdot}6 \times \frac{3^2}{1^2} = \mathbf{-122{\cdot}4\ eV}$

8(a).17. EXCITATION AND IONISATION POTENTIALS

Whenever an electron revolving in a stationary orbit of an atom absorbs some energy, two things may occur :

1. The electron may jump over to an outer orbit of higher energy. ***This process in which absorption of energy by an electron takes the electron from an inner orbit to some outer orbit of higher energy is called excitation. The atom is said to be in the excited state. The minimum accelerating potential which provides an electron energy sufficient to jump from the inner most orbit (ground state) to one of the outer orbits is called excitation potential or resonance potential.***

For example, in case of hydrogen atom, energies of electron in various orbits are :

$$E_1 = -\ 13{\cdot}6\ \text{eV},\ E_2 = -\ 3{\cdot}4\ \text{eV},\ E_3 = -\ 1{\cdot}51\ \text{eV and so on},\ E_\infty = 0.$$

∴ Energy required to raise an electron from ground state ($n = 1$) to first excited state ($n = 2$) is

$$E = E_2 - E_1 = -\ 3{\cdot}4 - (-\ 13{\cdot}6) = -\ 3{\cdot}4 + 13{\cdot}6 = 10{\cdot}2\ \text{eV}$$

The corresponding excitation potential = 10·2 volt

Similarly, energy required to raise an electron from ground state ($n = 1$) to second excited state ($n = 3$) is $E = E_3 - E_1 = -1{\cdot}51 - (-13{\cdot}6) = -1{\cdot}51 + 13{\cdot}6 = 12{\cdot}09$ eV

The corresponding excitation potential = 12·09 volt and so on .

Thus, excitation potential of an atom is not one. It can have many values, depending on the state to which the atom is excited.

2. If the energy supplied is so large that it can knock out an electron from the atom, the process is called **Ionisation.** ***Thus, ionisation is the phenomenon of removal of an electron from an atom. The minimum accelerating potential which would provide an electron energy sufficient just to remove it from the atom is called Ionisation potential.***

For example, total energy of electron in ground state of hydrogen atom = – 13·6 eV. To remove this electron from the hydrogen atom, + 13·6 eV energy is required.

∴ Ionisation energy of hydrogen atom = 13·6 eV, Ionisation potential of hydrogen atom = 13·6 volt.

The general expression for ionisation potential of an atom is $V = \frac{13{\cdot}6\, Z^2}{n^2}$ volt, where Z is the charge number of the atom and n is number of orbit from which electron is to be removed.

Note that for a given element in ground state, ionisation potential is fixed. But for different elements in ground state, ionisation potentials are different.

It should be clearly understood that when an element is not in ground state, ionisation energy of the element will not be unique. Ionisation energy of the given element will vary with the initial excited state of the element.

For example, if hydrogen atom is in first excited state, $E_2 = -3{\cdot}4$ eV. Therefore, its ionisation energy $= E_\infty - E_2 = 0 - (-3{\cdot}4)$ eV = 3·4 eV. Similarly, if hydrogen atom is in second excited state, its energy $E_3 = -1{\cdot}51$ eV. Therefore, its ionisation energy would be $E_\infty - E_3 = 0 - (-1{\cdot}51)$ eV = 1·51 eV.

8(a).18. X-RAY SPECTRA : CONTINUOUS AND CHARACTERISTIC

There are two types of X-ray spectra :

1. Continuous X-ray spectrum,
2. Characteristic X-ray spectrum.

(*a*) **Continuous X-ray spectrum** is that which consists of all wavelengths larger than a certain minimum wavelength, λ_{min}. The intensity of X-rays forming continuous spectrum varies with the voltage applied across the cathode and target of X-ray tube as shown in Fig. 8(a).17. This spectrum does not depend on nature of material of the target.

Consider an electron of initial KE = K_0 that interacts (collides) with one of the target atoms, as shown in Fig. 8(a).18. The electron may lose a small amount of energy ΔK, which would appear in the form of X-ray photon. This is radiated away from the site of collision.

If we ignore the energy transferred to the recoiling target atom, the energy of scattered electron would be ($K_0 - \Delta K$).

The scattered electron may have a second collision with a target atom generating another X-ray photon and so on. This electron scattering process continues till electron is approximately stationary. All the photons generated by such electron collisions form part of continuous X-ray spectrum.

FIGURE 8(a).17

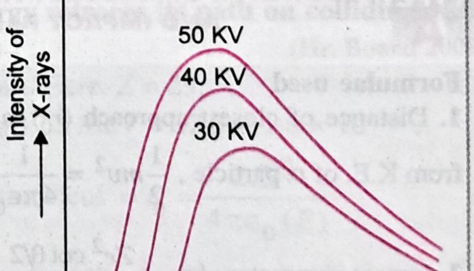
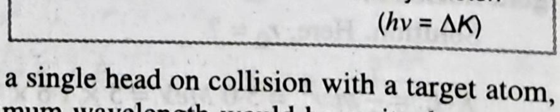


FIGURE 8(a).18



If incident electron were to lose all its initial KE (K_0) in a single head on collision with a target atom, a single X-ray photon of maximum energy/frequency or minimum wavelength would be emitted.

$$K_0 = h\nu_{max} = \frac{hc}{\lambda_{min}} \quad \therefore \lambda_{min} = \frac{hc}{K_0}$$

This is the cut off wavelength of continuous X-ray spectrum, which does not depend upon the target material.

(*b*) **Characteristic X-ray spectrum** is that which consists of a few particular wavelengths superposed on the continuous X-ray spectrum in the form of peaks as shown in Fig. 8(a).19. These wavelengths are characteristic of the target material of X-ray tube.

The peaks arise when an energetic electron strikes an atom of the target and knocks out one of the atom's deep lying electron, creating a **vacancy** or **hole,** in this shell (say *K* shell, $n = 1$). An electron in one of the outer shells having higher energy jumps to *K* shell, filling the vacancy/hole. During this jump, the atom emits a characteristic X-ray photon. If the jump is from *L* shell ($n = 2$), the emitted radiation is K_α line. If the jump is from *M* shell, the emitted radiation is K_β line and so on.

Similarly we obtain L_α, L_β lines when electron jumps to *L* shell from *M* shell and *N* shell respectively. Some of the spectral lines are shown in Fig. 8(a).20. Their wavelengths would depend upon nature of the target material of X-ray tube.

FIGURE 8(a).19

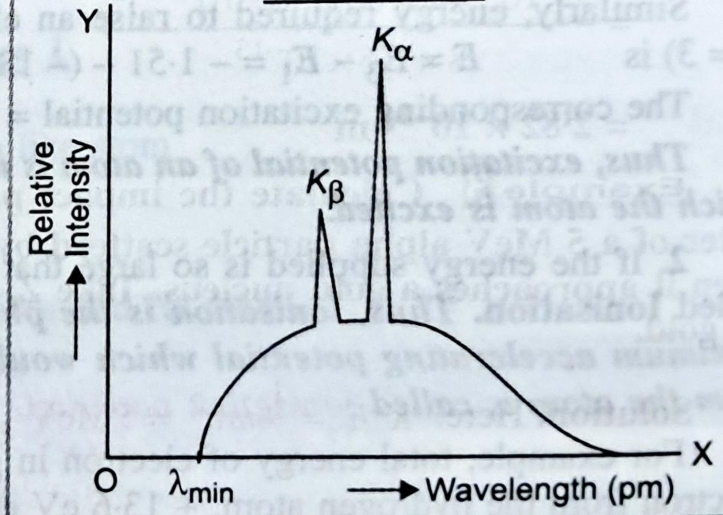
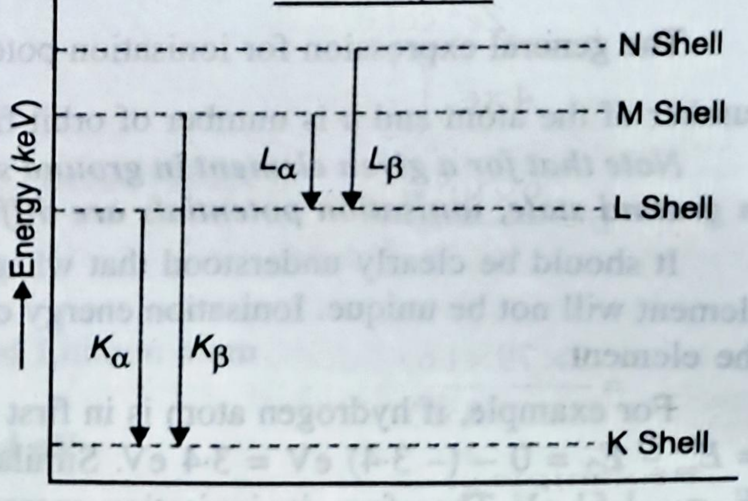


FIGURE 8(a).20



8(b)

NUCLEI

CHAPTER OVERVIEW

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8(b).1. INTRODUCTION

In the last chapter, we learnt that atomic nucleus was discovered in the year 1911 by Rutherford and his associates from the study of large angle scattering of alpha particles from thin metal foils.

Rutherford established that atomic nucleus is the central core of every atom which contains entire positive charge and more than 99·9% of the mass of the atom. Whereas the size of an atom is of the order of 10^{-10} m, the size of atomic nucleus is of the order of 10^{-15} m. Thus, most of the space around the nucleus of an atom is empty space.

In this chapter, we shall study the constituents of the nucleus and how they are held together. We shall also study size, mass, density and stability of nuclei. Finally, we shall have a look at the associated nuclear phenomena such as radioactivity, nuclear fission and nuclear fusion.

8(b).2. ATOMIC MASSES

For an insight into the nuclear phenomena, it is essential to determine accurately, the atomic mass of an element. The instrument designed for this purpose is called a **mass spectrometer.** Two commonly used mass spectrometers are due to Aston and Bainbridge.

The unit in which atomic and nuclear masses are measured is called **atomic mass unit** (a.m.u.) or **unified mass unit** (*u*).

***One atomic mass unit* or *unified mass unit* is defined as 1/12 th of the mass of an atom of $_6C^{12}$ isotope.**

As Avogadro's number = $6{\cdot}023 \times 10^{23}$, $\therefore$ Mass of $6{\cdot}023 \times 10^{23}$ atoms of C^{12} = 12 g

$$\text{Mass of one atom of } C^{12} = \frac{12}{6{\cdot}023 \times 10^{23}}\ \text{g}$$

$$\text{By definition,} \quad 1\ \text{a.m.u.} = \frac{1}{12} \times \text{mass of one atom of } C^{12}$$

$$\therefore \quad 1\ \text{a.m.u.} = \frac{1}{12} \times \frac{12}{6{\cdot}023 \times 10^{23}}\ \text{g} = 1{\cdot}66 \times 10^{-24}\ \text{g}$$

$$\boxed{1\ \text{a.m.u.} = 1{\cdot}66 \times 10^{-27}\ \text{kg}}$$

As an atom of $_6C^{12}$ contains 12 nucleons, therefore, *one a.m.u. represents the average mass of a nucleon and is denoted by u.*

In terms of this unit, mass of an electron (m_e) = 0·00055 u, mass of proton (m_p) = 1·0073 u mass of a hydrogen atom = $m_e + m_p$ = 1·0078 u, mass of a neutron (m_n) = 1·0086 u.

Atomic masses can also be expressed in terms of unit of energy, which is **electron volt.**

***One electron volt* is the energy gained by an electron, when accelerated through a potential difference of one volt.**

As work done = charge × potential

$$\therefore \quad 1\ \text{eV} = 1{\cdot}602 \times 10^{-19}\ (C) \times 1\ (V) = 1{\cdot}602 \times 10^{-19}\ \text{joule.}$$

$$1\ \text{MeV} = 1\ \text{million eV} = 10^6\ \text{eV} = 1{\cdot}602 \times 10^{-13}\ \text{joule}$$

Relation between a.m.u. and MeV

According to Einstein, mass energy equivalence is represented by $E = mc^2$

Taking $m = 1$. a.m.u. = $1{\cdot}66 \times 10^{-27}$ kg. and $c = 3{\cdot}0 \times 10^8$ m/s

we get, $E = (1{\cdot}66 \times 10^{-27})\,(3{\cdot}0 \times 10^8)^2\ \text{J} = 1{\cdot}49 \times 10^{-10}\ \text{J}$

$$\therefore \quad E = \frac{1{\cdot}49 \times 10^{-10}}{1{\cdot}6 \times 10^{-13}}\ \text{MeV} = 931{\cdot}25\ \text{MeV}$$

Hence, $\boxed{1\ \text{a.m.u.} \approx 931\ \text{MeV}}$, *which is used as a standard conversion.*

8(b).3. COMPOSITION OF NUCLEUS

The study of radioactive disintegrations indicates the emission of alpha, beta and gamma particles/rays to be of nuclear origin. Further, study of artificial radioactivity has revealed that many other particles like proton, neutron, alpha particles, beta particles and sub-particles like meson, positron, neutrino enter into the constitution of the nucleus in one way or the other. Based on these studies, several hypothesis have been put forward about the structure of the atomic nucleus. Prominent among them are :

1. Proton-Electron hypothesis, 2. Proton-Neutron hypothesis.

(a) Proton Electron hypothesis

This was put forward in the year 1930 to account for the emission of alpha and beta particles from the nuclei of radioactive elements.The positive charge in the nucleus is that of protons. A proton carries one unit of fundamental positive charge and is stable. An electron carries one unit of fundamental negative charge and its mass is negligible compared to the mass of proton.

According to proton electron hypothesis, the nucleus of an atom of mass number A and atomic number Z is made up of A protons and (*A–Z*) electrons. As every atom is electrically neutral, it must contain *Z* more electrons. They revolve around the nucleus in circular orbits. However, this hypothesis was rejected later in view of the **following problems :**

(*i*) According to de Broglie hypothesis and Heisenberg's uncertainty principle, if an electron is to exist inside the nucleus, it should possess energy ranging from 20 MeV to 200 MeV.
But the energy of electrons emitted during β-decay is at the most 2 to 3 MeV. Therefore, existence of electrons inside the nucleus is not justified on the basis of wave mechanics.

(*ii*) The observed values of nuclear spin or angular momentum of nuclei rule out the possibility of existence of electrons inside the nucleus.

(*iii*) Experimentally, the values of magnetic moments of nuclei are much smaller than the magnetic moments of electrons. Therefore, electrons cannot exist inside the nucleus.

(*iv*) The presence of a few electrons inside the nucleus and others revolving in orbits around the nucleus show dual role of electrons in the atomic structure, which is difficult to visualize.

Discovery of Neutron

A neutron is a neutral particle carrying no charge, and having mass equal roughly to the mass of a proton. This particle was discovered experimentally by **Chadwick** in the year 1932 :

(*i*) Chadwick observed that when beryllium was bombarded with alpha particles, some neutral radiations were emitted, which could knock out protons from light nuclei like helium, carbon and nitrogen.

(*ii*) Application of the principles of conservation of energy and momentum showed that neutral radiations from bombardment of beryllium could not be photons. Chadwick solved this puzzle by assuming that these neutral radiations consisted of hitherto unknown neutral particles – which were called neutrons.

Chadwick estimated the mass of a neutron being roughly equal to mass of a proton.

Whereas a free proton is stable ; a free neutron is unstable and has a mean life of 1000 second. Neutron is however, stable inside the nucleus.

Chadwick was awarded the 1935 Nobel Prize in Physics for his discovery of neutron.

(b) Proton-neutron hypothesis

This hypothesis was put forward by Heisenberg after the discovery of neutron by Chadwick.

According to this hypothesis, a nucleus of mass number *A* and atomic number *Z* contains *Z* protons, and (*A–Z*) neutrons. As an atom is electrically neutral, therefore, number of peripheral electrons must be equal to Z, the number of protons inside the nucleus.

This hypothesis accounted for all the discrepancies of proton electron hypothesis.

As is known, a neutron is a neutral particle carrying no charge. A proton carries a unit positive charge. Proton is slightly lighter than a neutron.

$$\text{mass of proton, } m_p = 1{\cdot}6729 \times 10^{-27}\text{ kg} = 1{\cdot}007825\text{ amu}$$

$$\text{mass of neutron, } m_n = 1{\cdot}6743 \times 10^{-27}\text{ kg} = 1{\cdot}008665\text{ amu}$$

A proton and a neutron may be regarded as two different states of the same particle, called **nucleon.** As the atom is electrically neutral, number of electrons in an atom is equal to the number of protons inside the nucleus.

Atomic Number *of an element is the number of protons present inside the nucleus of an atom* of the element. It is also equal to number of electrons revolving in various orbits around the nucleus of the atom. It is represented by Z.

Mass Number *of an element is the total number of protons and neutrons present inside the atomic nucleus* of the element. It is represented by A.

Thus, in an atom, number of protons $= Z$; number of electrons $= Z$

number of nucleons $= A$; number of neutrons $= (A - Z)$

A **Nuclide** is a specific nucleus of an atom, which is characterised by its atomic number Z, and mass number A. It is also called **nuclear species.**

It is represented as ${}_{Z}X^{A}$ where X is the chemical symbol of the species. For example, a gold nucleus is represented by ${}_{79}Au^{197}$. It contains 79 protons and 197 – 79 = 118 neutrons.

More about Proton and Neutron

Proton

Proton is a constituent particle of atomic nucleus. It carries unit positive charge $= 1{\cdot}6 \times 10^{-19}$ C and a mass equal to $1{\cdot}6729 \times 10^{-27}$ kg $= 1{\cdot}007825$ u. It has spin or intrinsic, angular momentum $= \frac{1}{2}\ (h/2\ \pi)$, where h is Planck's constant.

Proton is the lightest baryon with a magnetic moment of 2·79 nuclear magneton. Number of protons in a nucleus gives us *atomic number* of the nucleus.

Neutron

A neutron is a constituent particle of atomic nucleus. It carries no charge, and has a mass $= 1{\cdot}6743 \times 10^{-27}$ kg $= 1{\cdot}008665$ u, which is slightly more than the mass of a proton.

Inside the nucleus, a neutron is a stable particle. However, outside the nucleus, a neutron is unstable having a mean life time of 1000 s. It decays into a proton, an electron and antineutrino.

$$ {}_{0}n^{1} \longrightarrow {}_{1}H^{1} + {}_{-1}e^{0} + \bar{\nu} $$

Neutron has a spin (intrinsic angular momentum) equal to $\frac{1}{2}\ (h/2\ \pi)$, where h is Planck's constant. Magnetic moment of a neutron is – 1·91 nuclear magneton. Neutron has low ionising power.

As a neutron carries no charge, it is neither attracted nor repelled by the nucleus. Therefore, a neutron has a large penetrating power.

A fast moving neutron can be slowed down by certain materials like ordinary water, graphite, heavy water. These materials are called **moderators.** Slow neutrons, which are in thermal equilibrium with the molecules of the moderator are called **Thermal Neutrons.** Their energy is of the order of 0·025 eV. Thermal neutrons are most suitable for causing nuclear fission.

8(b).4. NUCLEAR SIZE

Experimental measurements show that ***volume of a nucleus is proportional to its mass number A.*** If R is the radius of the nucleus assumed to be spherical*, then its volume $\left(= \frac{4}{3} \pi R^3 \right) \propto A$

or $\qquad R \propto A^{1/3} \qquad$ or $\qquad \boxed{R = R_0 A^{1/3}} \qquad$ ...(1)

where R_0 is an empirical constant whose value is found to be $1{\cdot}2 \times 10^{-15}$ m.

As A is different for different elements, therefore, ***atomic nuclei of different elements have different sizes.***

---

*Though most of the nuclides are spherical, some of them are ellipsoidal, too.

8(b).5. NUCLEAR DENSITY

Density of nuclear matter is the ratio of mass of nucleus and its volume.

If m is average mass of a nucleon and R is the nuclear radius, then, mass of nucleus = mA, where A is the mass number of the element.

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$$\text{As density of nuclear matter} = \frac{\text{mass of nucleus}}{\text{volume of nucleus}} \quad \therefore \rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$\text{Thus,} \quad \boxed{\rho = \frac{3m}{4\pi R_0^3}} \qquad \text{...(2)}$$

DO YOU KNOW ?

Volume of a nucleus is directly proportional to mass number A of the nucleus. The volume does not depend upon separate number of protons and neutrons.

As m and R_0 are constants, therefore ***density ρ of nuclear matter is the same for all nuclei.***

Using $m = 1{\cdot}66 \times 10^{-27}$ kg,

$R_0 = 1{\cdot}2 \times 10^{-15}$ m

$$\text{we get,} \quad \rho = \frac{3 \times 1{\cdot}66 \times 10^{-27}}{4 \times 3{\cdot}14\,(1{\cdot}2 \times 10^{-15})^3} = \mathbf{2{\cdot}29 \times 10^{17}\ kg/m^3},$$

which is very large as compared to density of ordinary matter. For example, for water, $\rho \approx 10^3$ kg/m³ and for air, $\rho = 1{\cdot}293$ kg/m³. Hence matter in the nucleus is very densely packed.

RETAIN IN MEMORY

Note that sizes of nuclei are different, the heavier nuclei being bigger than the lighter ones. But density of nuclear matter is same for all nuclei. Value of nuclear density is far more greater than the atomic density.

IMPORTANT NOTE

High energy electron scattering experiments have established that distribution of matter inside the nucleus is not uniform. Density of atomic nucleus is maximum at the centre and it decreases as we move outwards from the centre of the nucleus.

Sample Problem 1 **What are the numbers of protons and neutrous in the nucleus of $_{92}U^{238}$?**

Sol. Here $_ZX^A = {}_{92}U^{238}$

Number of protons $= Z = \mathbf{92}$

Number of neutrons $= (A - Z) = 238 - 92 = \mathbf{146}$

Sample Problem 2 **Obtain approximately the ratio of nuclear radii of $_{26}Fe^{56}$ and $_{92}U^{238}$. What is the approximate ratio of their nuclear densities ?**

Sol. Here, $A_1 = 56$, $A_2 = 238$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{56}{238}\right)^{1/3} = (0{\cdot}235)^{1/3} = \mathbf{0{\cdot}617}$$

As nuclear density is same for all nuclei, therefore, $\dfrac{\rho_1}{\rho_2} = \mathbf{1}$

8(b).6. ISOTOPES

Isotopes ***of an element are the atoms of the element which have the same atomic number (Z) but different mass number (A).***

For example, $_1H^1$, $_1H^2$, $_1H^3$ are the isotopes of hydrogen. $_2He^3$, $_2He^4$, $_2He^6$ are the isotopes of helium. $_6C^{10}$, $_6C^{11}$, $_6C^{12}$, $_6C^{13}$, $_6C^{14}$ are the isotopes of carbon.

As isotopes of an element have the same atomic number, they contain ***same number of protons and same number of electrons.*** But as their atomic weights are different, they contain ***different number of neutrons.***

DO YOU KNOW ?

Isotope is a Greek word, which means same place. All isotopes of an element occupy the same place in Periodic Table of elements. This is because their atomic number (Z) is same.

Isotopes of an element have identical chemical properties. Their physical properties, however, differ.

All the known elements have one or more isotopes. The relative abundance of different isotopes differs from element to element. The atomic weight of an element is the weighted average of the masses of all its isotopes based on the occurrence of each isotope in nature. For example, neon has two isotopes of masses 20 and 22 which occur in the ratio, 9 : 1.

$$\therefore \text{ Average atomic weight of neon } = \frac{20\times9+22\times1}{(9+1)} = \frac{180+22}{10} = 20{\cdot}2$$

On the contrary, *mass number A is always an integer, as it represents the number of nucleons in the nucleus.*

8(b).7. ISOBARS

Isobars ***are the atoms of different elements which have the same mass number (A), but different atomic numbers (Z).***

Isobars contain different number of protons, different number of electrons and also different number of neutrons. Only the total number of nucleons in them is the same.

For example, $_{11}Na^{22}$ and $_{10}Ne^{22}$ are isobars. Similarly, $_{20}Ca^{40}$ and $_{18}Ar^{40}$ are isobars and $_{17}Cl^{37}$, $_{16}S^{37}$ are also isobars.

Similarly, $_1H^3$ and $_2He^3$ are isobars. Also, $_6C^{14}$ and $_7N^{14}$ are isobars.

The chemical properties of isobars are widely different. Their physical properties may be identical. The isobars occupy different places in the periodic table.

8(b).8. ISOTONES

Isotones ***are the nuclides which contain the same number of neutrons i.e. in their case, (A–Z) = N is the same. Their atomic number (Z) and mass number (A) are different.***

For example, $_{17}Cl^{37}$ and $_{19}K^{39}$ are isotones. This is because number of neutrons in chlorine = 37 – 17 = 20 and number of neutrons in potassium = 39 – 19 = 20.

Similarly, $_1H^3$ and $_2He^4$ are isotones. Also, $_8O^{16}$ and $_6C^{14}$ are isotones.

Some other examples of isotones are :

$_4Be^9$ and $_5B^{10}$; $_6C^{13}$ and $_7N^{14}$; $_{11}Na^{23}$ and $_{12}Mg^{24}$.

Sample Problem **Choose one pair of isotopes, isobars and isotones of the following :**

$_{18}Ar^{40}$; $_{17}Cl^{37}$; $_{11}Na^{23}$; $_{17}Cl^{35}$; $_{12}Mg^{24}$; $_{20}Ca^{40}$

Sol. *(i)* $_{17}Cl^{37}$ and $_{17}Cl^{35}$ are isotopes with same $Z = 17$ and different $A = 37$ & 35

(ii) $_{18}Ar^{40}$ and $_{20}Ca^{40}$ are isobars with same $A = 40$ and different $Z = 18$ & 20

(iii) $_{11}Na^{23}$ and $_{12}Mg^{24}$ are isotones with same number of neutrons = (A – Z) = 12

8(b).9. MASS–ENERGY RELATION

Einstein was the first to establish the equivalence of mass and energy through the famous relation

$$\boxed{E = mc^2}$$

where $c = 3 \times 10^8$ m/s = speed of light in vacuum.

The relation means that when a certain mass m disappears, an equivalent amount of energy E appears and vice-versa.

The mass of a particle measured in a frame of reference in which the particle is at rest is called its **rest mass,** usually denoted by m_0. The rest mass energy of particle would be $m_0 c^2$, which is enormously large on account of large value of c. If T is kinetic energy of the particle, then its total energy

DO YOU KNOW ?

The relation $E = mc^2$ applies equally to chemical reactions. But the magnitude of mass defect in the chemical reactions is about million times smaller than that in nuclear reactions. Therefore, it is hardly noticed.

$$\boxed{E = mc^2 = \text{rest mass energy} + \text{K.E.} = m_0 c^2 + T} \quad ...(3)$$

Here, m is called **effective mass** of the particle, when it is moving. Clearly, $m > m_0$.

From eqn. (3), $T = mc^2 - m_0 c^2 = (m - m_0) c^2$

$$\boxed{T = (\Delta m)\, c^2}$$

Thus $K.E.$ of a particle = change in mass of particle × (speed of light in vacuum)^2.

In nuclear and elementary particle interactions, both the conversion of *mass into energy* and the conversion of *energy into mass* take place. Therefore, the two classical laws of conservation of mass and conservation of energy have been unified into one law–***the law of conservation of mass energy. According to this law, the sum of the mass-energy of a system of particles is the same, before and after an interaction.***

This law of conservation of mass energy has been verified in all nuclear reactions.

Sample Problem **Express 150 microgram mass into equivalent energy in electron volt.**

Sol. Here, $m = 150$ microgram $= 150 \times 10^{-6}$ g $= 150 \times 10^{-9}$ kg, $E = ?$

From $E = mc^2 = (150 \times 10^{-9})(3 \times 10^8)^2 = 1{\cdot}35 \times 10^{10}$ J

$$\therefore \quad E = \frac{1{\cdot}35 \times 10^{10}}{1{\cdot}6 \times 10^{-19}}\ \text{eV} = \mathbf{0{\cdot}843 \times 10^{29}\ eV}$$

8(b).10. NUCLEAR BINDING ENERGY

(a) Concept of binding energy

An atomic nucleus is a stable structure. Therefore, nucleons in every nucleus are bound together with short range interacting forces, called nuclear forces. A definite amount of work has to be done to separate the nucleons from the nucleus to such a distance that there is no interaction between them. This work done is a measure of binding energy of the nucleus. Thus,

> ***Binding energy of a nucleus is the energy with which nucleons are bound in the nucleus. It is measured by the work required to be done to separate the nucleons an infinite distance apart from the nucleus, so that they may not interact with each other.***

The origin of nuclear binding energy has been explained on the basis of Einstein's theory of mass energy equivalence. It is found that rest mass of a nucleus is always slightly less than the sum of the rest masses of free neutrons and protons composing the nucleus. This is as if certain mass disappears in the formation of the nucleus. ***This difference between the sum of the masses of neutrons and protons forming a nucleus and mass of the nucleus is called* mass defect.** It is this mass defect which appears in the form of binding energy, responsible for binding the nucleons together in the nucleus.

(b) Expression for Nuclear Binding Energy

In a nucleus ${}_ZX^A$, Z = charge number = number of protons.

A = mass number = number of protons plus the number of neutrons

$\therefore$ number of neutrons = $(A - Z)$

Let m_p = mass of a proton,

m_n = mass of a neturon

m_N = mass of nucleus, ${}_Z X^A$

$$\therefore \quad \boxed{\text{Mass defect, } \Delta m = [Zm_p + (A - Z)\, m_n - m_N]} \qquad \text{...(3)}$$

Using Einstein's mass energy equivalence, Binding Energy = $\Delta m.c^2$

$$\text{B.E.} = [Zm_p + (A - Z)\, m_n - m_N]\, c^2 \qquad \text{...(4)}$$

where c is velocity of light in vacuum.

If m is the mass of the atom ${}_Z X^A$ containing Z electrons each of mass m_e, then we can write

$$m\left({}_Z X^A\right) = m_N\left({}_Z X^A\right) + Zm_e - \frac{\text{B.E. of electrons}}{c^2} \qquad \text{...(5)}$$

For hydrogen atom of mass m_H containing one electron, we can write

$$m_{\text{H}} = m_p + m_e - \frac{\text{B.E. of one electron}}{c^2} \qquad \text{...(6)}$$

Now, the binding energy of electrons is much smaller ≈ eV to keV, as compared to the rest mass energy of a nucleon ≈ 10^3 MeV, therefore, we can safely ignore in (5) and (6), the *terms involving binding energy of electrons.*

$\therefore$ Eqns. (5) and (6) reduce to

$$m\left({}_Z X^A\right) = m_N\left({}_Z X^A\right) + Zm_e \qquad \text{...(7)}$$

and

$$m_H = m_p + m_e \qquad \text{...(8)}$$

Rewriting eqn. (4), as

$$\text{B.E.} = [Zm_p + Zm_e + (A - Z)\, m_n - m_N - Zm_e]\, c^2$$

$$\text{B.E.} = [Z\,(m_p + m_e) + (A - Z)\, m_n - (m_N + Zm_e)]\, c^2$$

Using (7) and (8), we get

$$\boxed{\text{B.E.} = [Zm_H + (A - Z)\, m_n - m\,({}_ZX^A)]\, c^2} \qquad \text{...(9)}$$

This is the required expression for B.E.

(c) Average Binding Energy per nucleon of a nucleus

It is the average energy we have to spend to remove a nucleon from the nucleus to infinite distance. It is given by total binding energy divided by the mass number of the nucleus.

The variation of average B.E. per nucleon with mass number A is shown in Fig. 8(b).1. This curve reveals the following facts :

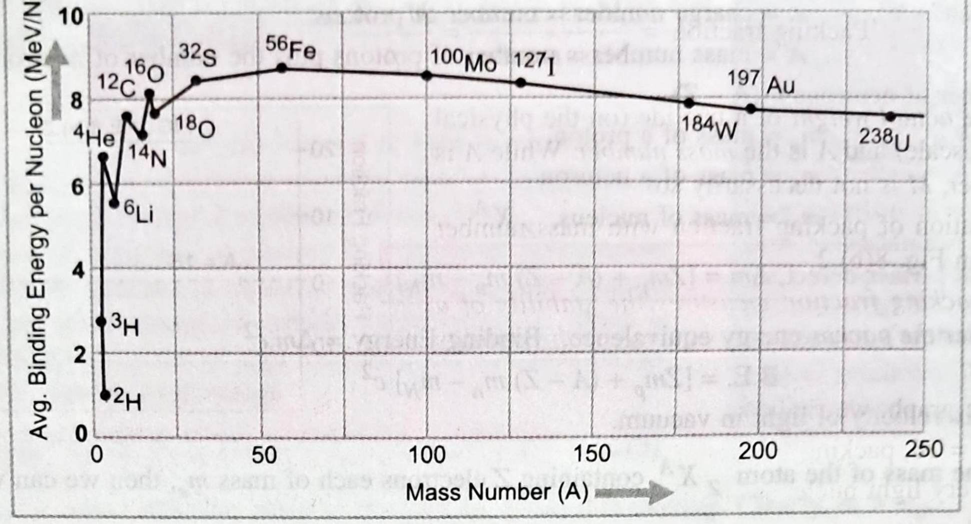
1. Average B.E./nucleon for light nuclei like ${}_1H^1$, ${}_1H^2$, ${}_1H^3$, is small.

2. For mass numbers ranging from 2 to 20, there are sharply defined peaks corresponding to ${}_2He^4$, ${}_6C^{12}$, ${}_8O^{16}$ etc. The peaks indicate that these nuclei are relatively more stable than the other nuclei in their neighbourhood.

3. The B.E. curve has a broad maximum in the range $A = 30$ to $A = 120$ correspoding to average binding energy per nucleon = 8·5 MeV. The peak value of the maximum is 8·8 MeV/N for ${}_{26}Fe^{56}$.

4. As the mass number increases, the B.E./nucleon decreases gradually falling to about 7·6 MeV per nucleon for ${}_{92}U^{238}$. The decrease may be due to Coulomb repulsion between the protons. The heavy nuclei are therefore, relatively less stable.

FIGURE 8(b).1



(d) Importance of Binding Energy Curve

(*i*) When we move from the heavy nuclei region to the middle region of the plot, we find that there will be a gain in the overall binding energy and hence release of energy. This indicates that *energy can be released when a heavy nucleus (A ≈ 240) breaks into two roughly equal fragments.* This process is called **Nuclear Fission.**

(*ii*) Similarly, when we move from lighter nuclei to heavier nuclei, we again find that there will be gain in the overall binding energy and hence release of energy. This indicates that *energy can be released when two or more lighter nuclei fuse together to form a heavy nucleus.* This process is called **Nuclear Fusion.**

Hence, the importance of binding energy curve is that it led us to the possibility of release of nuclear energy by nuclear fission and nuclear fusion. These phenomena of nuclear fission and nuclear fusion were actually discovered later.

RETAIN IN MEMORY When mass defect (Δm) is in a.m.u., energy released in MeV can be calculated directly by multiplying mass defect with 931 (MeV). There is no need to apply ; $E = (\Delta m)\, c^2$.

Sample Problem **Calculate mass defect, binding energy and binding energy per nucleon for a lithium nucleus ($_3Li^7$) taking its mass = 7·000000 a.m.u., Mass of proton = 1·007825 a.m.u. and mass of neutron = 1·008665 a.m.u. Take 1 a.m.u = 931·5 MeV.**

Sol. Here in $_3Li^7$, number of protons = 3,

number of neutrons = 7 – 3 = 4

$\therefore$ mass defect, $\Delta m = 3\, m_p + 4\, m_n - M$

$$\Delta m = 3 \times 1{\cdot}007825 + 4 \times 1{\cdot}008665 - 7{\cdot}000000$$

$$= 7{\cdot}058135 - 7{\cdot}000000 = \mathbf{0{\cdot}058135\ amu}$$

$$\text{Total BE} = 0{\cdot}058135 \times 931{\cdot}5\ \text{MeV} = 54{\cdot}153\ \text{MeV}$$

$$\text{BE/nucleon} = \frac{54{\cdot}153}{7} = \mathbf{7{\cdot}736\ MeV/N}$$

DO YOU KNOW ?

The binding energy does not reside in the nucleus. It is a measure of how well the nucleons are held together in the nucleus. Infact, binding energy is the total energy that has to be transferred to the nucleons to move them to infinite separations from the nucleus.

8(b).11. PACKING FRACTION

Packing fraction ***of a nucleus is defined as the mass excess per nucleon.***

Mass excess is the difference between the mass of a nucleus and mass number of the nucleus. Thus,

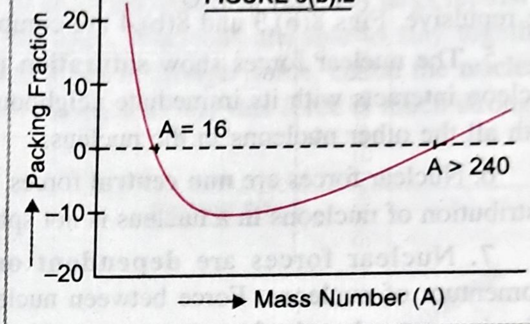
$$\boxed{\text{Packing fraction} = \frac{\text{Mass excess}}{\text{Mass number}} = \frac{M - A}{A}} \qquad ...(10)$$

where M is the *actual weight* of a nuclide (on the physical atomic weight scale) and A is the *mass number.* While A is a whole number, M is not necessarily so.

The variation of packing fraction with mass number (A) is shown in Fig. 8(b).2.

Infact, *packing fraction measures the stability of a nucleus. Smaller the packing fraction, larger is the stability of the nucleus.*

FIGURE 8(b).2



From the graph, we find that

(*i*) for $A = 16$, packing fraction is zero.

(*ii*) for very light nuclei ($A < 16$), packing fraction is large. Therefore, they are unstable.

(*iii*) As A increases beyond $A = 16$, packing fraction becomes negative, decreases with increasing mass number. Beyond certain value of mass number, packing fraction starts increasing till it becomes zero. At about $A > 240$, packing fraction becomes large. Thus heavy nuclei are unstable.

Clearly, nuclei of intermediate mass numbers are relatively stable.

8(b).12. NUCLEAR FORCES

We know that the force that determines the motion of atomic electrons is the *Coulomb force* of attraction between electrons and the nucleus. In the previous article, we noted that for nuclei of average mass, binding energy per nucleon is about 8 MeV, which is pretty large. Therefore, to bind a nucleus, there must be very strong attractive forces. Such forces must overcome repulsion between positively charged protons in the nucleus ; and bind both protons and neutrons into tiny nuclear volume ($\approx 10^{-45}$ m^3). These forces are called nuclear forces.

> ***Nuclear forces** are the strong forces of attraction which hold together the nucleons (neutrons and protons) in the tiny nucleus of an atom, inspite of strong electrostatic forces of repulsion between protons.*

These forces are very complex in nature. Some of the ***important characteristics*** of these forces are:

1. Nuclear forces act between a pair of neutrons, a pair of protons and also between a neutron, proton pair, with the *same strength.* This shows that **nuclear forces are independent of charge.**

2. Nuclear forces are the **strongest forces in nature.** The magnitude of nuclear forces is 100 times that of electrostatic forces and 10^{38} times that of gravitational forces between nucleons. That is why nucleons are held together in a nucleus inspite of electrostatic force of repulsion between protons.

3. The nuclear forces are **very short range forces.** They are operative upto distances of the order of a few fermi, *i.e.,* they operate only within the nucleus.

4. The **variation of nuclear forces with the distance** between nucleons is not known exactly. However, from Fig. 8(b).3, we observe that

(*i*) Nuclear forces are negligible, when distance between nucleons is more than 10 fermi.

(*ii*) when nucleons are brought closer, nuclear force of attraction develops which goes on increasing rapidly with decreasing distance. However *nuclear forces do not obey inverse square law.*

(*iii*) When distance between nucleons becomes less than 0·8 fermi, *the nuclear forces become strongly repulsive.*

Fig. 8(b).4 shows a rough plot of *potential energy between a pair of nucleons* with distance (r) between them.

The potential energy is *minimum* at a distance $r_0 = 0{\cdot}8$ fm. At this distance, force between nucleons is zero. For distances larger than 0·8 fm, netative P.E. goes on decreasing. The nuclear forces are attractive. For distances less than 0·8 fm ; negative P.E. decreases to zero and then becomes positive. The nuclear forces are repulsive. Figs 8(b).3 and 8(b).4 are compatible.

FIGURE 8(b).3

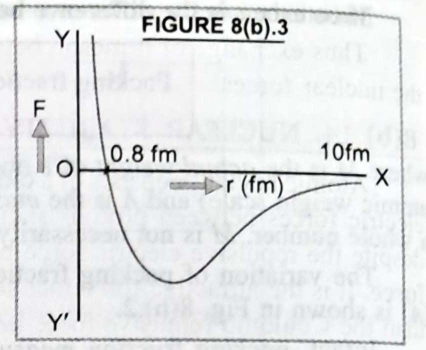
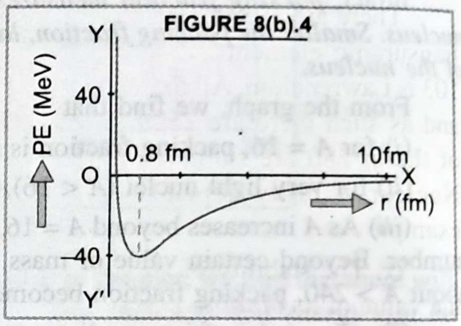


FIGURE 8(b).4



5. The nuclear forces show **saturation properties** *i.e.* each nucleon interacts with its immediate neighbours only, rather than with all the other nucleons in the nucleus.

6. Nuclear forces are **non central** forces. This shows that the distribution of nucleons in a nucleus is not spherically symmetric.

7. Nuclear forces are **dependent on spin** or angular momentum of nucleons. Force between nucleons having parallel spins is greater than the force between nucleons having antiparallel spins.

8. Nuclear forces are **due to exchange of π mesons** between the nucleons, as detailed in Art. 8(b).13. That is why they are called **Exchange Forces.**

RETAIN IN MEMORY Nuclear forces are the strongest attractive forces between nucleons – which are independent of charge, are short range ; non central, non conservative forces and not obeying inverse square law.

8(b).13. NATURE OF NUCLEAR FORCES

A Japanese Physicist Yukawa postulated in the year 1936 that *nuclear forces between nucleons arise on account of continuous exchange of particles called mesons between the nucleons.* Later on, these particles were detected experimentally. They were found to have a mass about 270 times the mass of an electron. Three types of mesons were detected. Mesons carrying no charge were called neutral pi meson (π^0). Mesons carrying unit positive charge each were called positive π meson (π^+) and mesons carrying unit negative charge each were called negative π meson (π^-).

According to meson theory or Yukawa theory of nuclear forces,

(*i*) All nucleons consist of identical cores surrounded by a pulsating cloud of π meson.

(*ii*) Mesons or pions may be neutral (π^0) or carry either charge (π^+ or π^-).

(*iii*) The difference between a proton and a neutron is essentially in the composition of their respective meson clouds.

(*iv*) The force between neutron and proton is due to exchange of charged meson between them, *i.e.*, $p^+ + \pi^- = n^0$

$$n^0 + \pi^+ = p^+$$

DO YOU KNOW ?

The present view is that the nuclear force that binds protons and neutrons in the nucleus is not a fundamental force of nature. Rather, nuclear force is a secondary effect of the strong force that binds *quarks* together to form neutrons and protons. Infact, when protons and neutrons are subjected to collisions at energies of many GeV (= 10^9 eV), their behaviour is explained by assuming that neutrons and protons are made up of quarks. Like electrons, quarks are thought to be indivisible.

Thus proton and neutron continuously exchange their nature by absorbing and emitting π mesons. Thus

$$n^0 \longrightarrow p^+ + \pi^- \quad \text{and} \quad p^+ \longrightarrow n^0 + \pi^+$$

(*v*) The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them, *i.e.*, $p \longrightarrow p' + \pi^0$ or $p + \pi^0 \longrightarrow p'$

and $n \longrightarrow n' + \pi^0$ or $n + \pi^0 \longrightarrow n'$

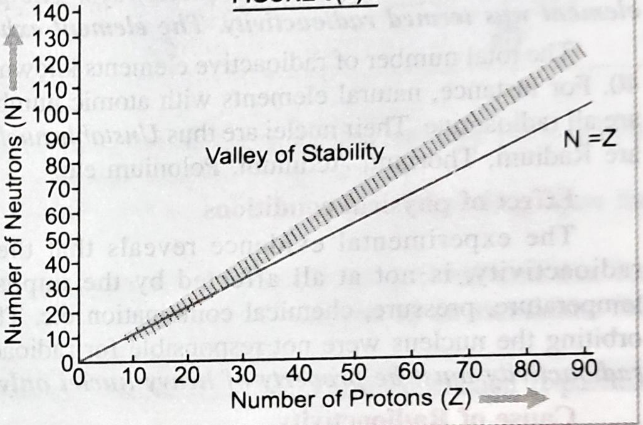
In both the cases, a proton or a neutron is converted into a new state after emitting or absorbing the π^0 meson.

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

8(b).14. NUCLEAR STABILITY

Atomic nucleus consists of a closely packed collection of protons and neutrons. The very large repulsive electric forces between protons should cause the nucleus to fly apart. Nucleons are able to stay together despite the repulsive electric force between protons, because of a much stronger force, called the **nuclear force.** It is the attractive force that acts between all nucleons in the nucleus and this force is much stronger than the Coulomb repulsive force between protons.

FIGURE 8(b).5



There are 103 different elements including the transuranic elements. $Z = 1$ is hydrogen and $Z = 103$ is Lawrencium. All these elements have isotopes and as such there are 2500 different nuclides. Out of these, 280 are stable and the rest are unstable. Nuclear stability depends on many factors. For example :

(*i*) The stability of a nucleus is determined by the value of its ***binding energy per nucleon.*** Higher the B.E/nucleon, more stable is the nucleus.

(*ii*) The stability of a nucleus is also determined by ***its neutron to proton ratio.*** A plot of neutron number (N) and proton number (atomic number Z) for stable nuclides is shown in Fig. 8(b).5. The solid line in the plot shows the locations of nuclei that have an equal number of protons and neutrons ($N = Z$). Note that only light nuclei are on this line *i.e.* light nuclei are stable only if they contain about the same number of protons and neutrons. ***Heavy nuclei, on the other hand, are stable only when they have more neutrons than protons.*** Thus, heavy nuclei are *neutron rich* compared to lighter nuclei. This is as if *neutrons help to stabilize a nucleus.* This is easily understood. More is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore, more neutrons are needed to provide the strong attractive forces necessary to keep the nucleus stable. The long narrow region in Fig. 8(b).5, which contains the cluster of short lines representing stable nuclei is referred to as the **valley of stability.**

(*iii*) The stability of a nuclide is also determined by the consideration whether it *contains an even or odd number of protons and neutrons.* A detailed study shows that stable nuclei often contain even number of protons or neutrons or both as listed in Table 8(b).1.

TABLE 8(b).1. Even/odd Number of protons and neutrons in stable nuclei

Number of stable nuclei	Number of protons (Z)	Number of neutrons (N)
165	even	even
57	even	odd
53	odd	even
6	odd	odd

DO YOU KNOW ?

The stability of a nuclide is intimately connected to the relative number of neutrons and protons in that nuclide. For small nuclei to be stable, the number of protons must be roughly equal to number of neutrons. However, when number of protons increases, more neutrons are needed to maintain stability.

8(b).15. RADIOACTIVITY

Natural radioactivity is a ***spontaneous and self disruptive activity*** exhibited by a number of heavy elements occuring in nature. The word spontaneous means shear out of nature of the element and self disruptive means that the element disintegrates itself, *i.e.*, no external provocation influences the emission*.

> ***Radioactivity is, therefore, the property by virtue of which a heavy element disintegrates itself without being forced by any external agent to do so.***

The phenomenon was ***discovered by a French Physicist, Henry Becquerel in 1896.*** He observed that uranium salts possessed a peculiar property of affecting a photographic plate even when the plate was in a light proof package. This he thought, must be due to certain active radiations emitted by uranium salts. These radiations were called *Becquerel* rays. ***The phenomenon of emission of active radiations by an element was termed radioactivity. The element exhibiting this property was called radioactive element.***

The total number of radioactive elements known at present is about 40. For instance, natural elements with atomic number greater than 82 are all radioactive. Their nuclei are thus *Unstable nuclei.* Some examples are Radium, Thorium, Actinium, Polonium etc.

DO YOU KNOW ?

The total number of isotopes of all elements exceeds 2500 and most of them are radioactive leaving only about 10% of known isotopes, which are stable.

Effect of physical conditions

The experimental evidence reveals that the phenomenon of radioactivity, is not at all affected by the imposed conditions of temperature, pressure, chemical combination etc. Therefore, electrons orbiting the nucleus were not responsible for radioactivity. Hence, ***the radioactivity must be property of heavy nuclei only.***

Cause of Radioactivity

Emission of active radiations indicates that parent nucleus is **unstable.** We notice from the binding energy curve [Fig. 8(*b*).3] that *BE*/nucleon goes on decreasing, though slowly, for nuclei beyond $A > 140$. This indicates that stability decreases as we move towards heavier nuclei. The heavy nuclei have larger atomic number Z and hence contain larger number of protons. The mutual repulsion of protons reduces the binding effect of nuclear forces. This is the main cause of relative instability of heavy nuclei. Radioactivity results from this instability.

8(b).16. THREE TYPES OF RADIATIONS

Experimental investigations by Rutherford and others have proved that the radiations emitted by radioactive elements consist of three kinds, depending upon their ability to penetrate matter. One kind of radiations ***with least penetrating power (i.e. the one which would stop first) was named Alpha rays.***

The other kind with ***a comparatively larger penetrating power was called Beta rays.***

The last kind with ***maximum penetrating power was called Gamma rays.*** It should be clearly noted that all the three types of radiations need not necessarily be emitted by one radioactive element.

NATURE OF α, β, γ-RAYS

A number of attempts were made to study the nature of α, β and γ-rays. One of the methods used is shown in Fig. 8(b).6. The sample of a radioactive element (say, radium) is placed in a small cavity drilled in a lead block. The lead block checks all the α, β and most of the γ-rays, except those through the opening. The radiations coming out of the cavity are subjected to an electric field provided by two plates as shown in Fig. 8(b).6(a). ***The α-rays are deflected through smaller angles towards the negative plate. The β-rays are deflected through larger angles towards the positive plate. The γ-rays remain undeflected.*** As opposite charges attract each other, it was concluded that α-rays consist of a stream of positively charged particles, whereas β-rays consist of a stream of negatively charged particles. Since γ-rays were undeflected they could be *waves* or *uncharged particles.* Further analysis failed to detect any charge on γ rays and showed that they have no mass. γ-rays, therefore, were thought to be waves. Exactly same results were obtained when these radiations were subjected to the action of a magnetic field, [Fig. 8(b).6(b)].

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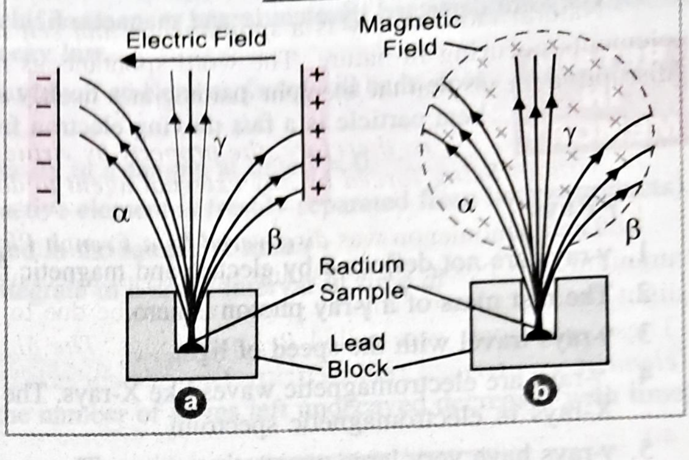
*Also, the emission cannot be controlled by physical or chemical means.

Some of the ***important properties of the three types of radiations*** (α, β and γ-rays) are discussed below :

FIGURE 8(b).6



α-Rays

1. ***An α-particle carries double the positive charge of proton,*** which is equal to the charge on the helium nucleus.
2. ***Mass of an α-particle is roughly four times that of hydrogen atom i.e.*** it is equal to the mass of the helium nucleus.

   The above two properties establish that an ***α-particle is equivalent to helium nucleus*** (or a helium atom which has lost its two orbital electrons *i.e.* **a doubly ionised helium atom**).
3. The velocity of α-particles ranges between $1{\cdot}4 \times 10^7$ ms^{-1} to $2{\cdot}1 \times 10^7$ ms^{-1}, depending upon the source emitting it.
4. Because of large mass, the penetrating power of α-particles is very small, it being 1/100 time that due to β-rays and 1/10,000 time that due to γ-rays. α-particles can be easily stopped by an aluminium foil only 0·02 mm thick. Air of thickness 7·0 cm absorbs them completely.
5. Because of large mass and large velocity, α-particles have large ionising power. Each α-particle produces about 20000 ion pairs before being absorbed.
6. The range of α-particles in air (distance through which they can travel in air) depends upon the radioactive source producing it. At normal pressure in air, the range of α-particles varies from 3 to 8 cm.
7. α-particles produce fluorescence in certain substances, like barium-plantinocyanide and zinc- sulphide.
8. α-particles affect photographic plate slightly.
9. α-particles are deflected by electric and magnetic fields.
10. α-particles are scattered while passing through thin metal foils. Most of the α-particles are scattered at small angles, but a few of them are scattered at an angle more than 90° also.
11. α-particles cause burns on human body.
12. α-particles on being stopped, produce heating effect.

β-Rays

1. A β-particle carries $1{\cdot}6 \times 10^{-19}$ C of negative charge, *i.e.* unit negative charge, which is the charge on an electron.
2. The rest mass of β-particle is $9{\cdot}1 \times 10^{-31}$ kg, which is the same as that of electron.
3. The velocity of β-particles ranges from 33% to 99% of the velocity of light.

   The above properties establish that β **particles are fast-moving electrons.**
4. Because of small mass, the penetrating power of β-particles is very large. They can easily pass through a few millimeter of aluminium.
5. The β-particles ionise the gas through which they pass, but their ionising power is only 1/100th that of α-particles.
6. The range of β-particles in air is several metres.
7. The β-particles can also produce flourescence in certain substances like barium platinocyanide and zinc sulphide.

8. They affect a photographic plate.
9. They are deflected by electric and magnetic fields, showing that they carry negative charge.

RETAIN IN MEMORY

Note that an alpha particle is a doubly ionised helium atom or a helium nucleus. And a beta particle is a fast moving electron from the nucleus.

γ-Rays

1. γ-rays are not deflected by electric and magnetic fields, showing that they do not carry any charge.
2. The rest mass of a γ-ray photon is zero.
3. γ-rays travel with the speed of light.
4. γ-rays are electromagnetic waves like X-rays. The wavelength of γ-ray photon is smaller than that of X-rays in electromagnetic spectrum.
5. γ-rays have very large penetrating power. They can pass through several centimetre of iron and lead.
6. γ-rays have got small ionising power.
7. γ-rays can produce fluorescence in a substance like willimite.
8. γ-rays affect a photographic plate more than β-particles.
9. γ-rays can knock out electrons from the surface of a metal, on which they may fall.
10. γ-rays can cause nuclear reactions.

NOTE: Ionizing Power and Penetrating Power

As is known, an ion is an atom with an overall charge. Charge is positive when some electrons are removed from a neutral atom and charge is negative, when some electrons are added to a neutral atom.

Alpha particles are moving slowly. They have time to interact with nearly all atoms in their path. The positive charge on alpha particles attracts electrons from atoms in their path and ionise them, losing energy quickly. Thus, α-particles have high ionizing power but poor penetrating power.

Beta particles are repelled by electrons in atoms. This repulsion causes β-particles to bounce between the atoms. Collisions with atoms may cause ionization of some of the atoms. Therefore, β particles lose energy slowly. Their penetrating power is higher than that of alpha, but ionizing power is smaller than that of alpha particles.

γ-rays have no charge. Therefore, their collisions occur only when a nucleus or electron is directly in their path. This is much less likely to happen because of large empty space in an atom. Therefore, gamma rays have very low ionizing power, and very high penetrating power.

8(b).17. LAWS OF RADIOACTIVE DISINTEGRATION

Soddy and Rutherford, from their experimental study regarding the radioactive disintegration formulated the following laws, known as laws of radioactive disintegration :

1. *Radioactivity is a spontaneous process which does not depend upon external factors like temperature, pressure etc. i.e. a radioactive element is in a state of disintegration which depends upon law of chance. It means it is impossible to predict which particular atom of the radioactive element will disintegrate in a given time interval.*

2. *During disintegration of an atom, either an α-particle or a β-particle is emitted. Both of these particles are never emitted simultaneously. Also at a time, an atom will not emit more than one α-particle or more than one β-particle. γ-rays emission follows the emission of α or β particle.*

3. *The emission of α-particle from an atom will change it, into a new atom whose charge number is reduced by two and mass number is reduced by four.*

4. *The emission of a β-particle from an atom will change it, into a new atom whose charge number is raised by one, without any change in its mass number.*

5. ***The number of atoms disintegrated per second (i.e. rate of disintegration of radioactive atoms) at any instant is directly proportional to the number of radioactive atoms actually present in the sample at that instant.*** This is also known as **radioactive decay law.**

Thus greater the number of atoms present in a sample of an element, faster will be its decay and vice-versa.

Mathematical Form of Radioactive Decay Law

Let N_0 = total number of atoms present originally in a sample at time $t = 0$.

($t = 0$ refers to the time when the radioactive element is freshly separated from its by-products).

N = total number of atoms left undecayed in the sample at time t

dN = a small number of atoms that disintegrate in a small interval of time dt

∴ Rate of disintegration of the element

$$\boxed{R = -\frac{dN}{dt}}$$ (minus sign indicates that the number of atoms left undecayed decreases with time)

According to radioactive decay Law, $-\frac{dN}{dt} \propto N$ ∴ $\boxed{R = -\frac{dN}{dt} = \lambda N}$ ...(11)

where λ is a constant of proportionality and is called the **disintegration constant** or the **decay constant**.

Equation (11) can be written as $\frac{dN}{N} = -\lambda\, dt$

Integrating both sides, we get, $\int \frac{dN}{N} = -\lambda \int dt$ or $\log_e N = -\lambda t + C$ ...(12)

where C is a constant of integration.

At $t = 0, N = N_0$

∴ From (12), $\log_e N_0 = \lambda \times 0 + C$

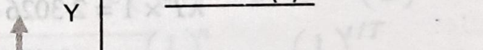
or $C = \log_e N_0$ ...(13)

Put in (12), $\log_e N = -\lambda t + \log_e N_0$

∴ $\log_e N - \log_e N_0 = -\lambda t$ or $\log_e \frac{N}{N_0} = -\lambda t$

i.e. $\frac{N}{N_0} = e^{-\lambda t}$ ∴ $\boxed{N = N_0\, e^{-\lambda t}}$ ...(14)

FIGURE 8(b).7



Equation (14) shows that the *radioactive decay is exponential* as shown in Fig. 8(b).7.

From Fig. 8(b).7, we find that $N = 0$ only at $t = \infty$. This is also clear from eqn. (14).

DO YOU KNOW ?

One milligram sample of $_{92}U^{238}$ contains $2{\cdot}5 \times 10^{18}$ atoms, which have existed without decaying since they were created – well before the formation of our solar system. During any given one second, only 12 of these nuclei in our sample will decay by emitting an alpha particle. And, there is absolutely no way to predict whether any particular nucleus in the sample will decay in the next second. All nuclei in the sample have the same probability/chance of decay.

8(b).18. DISINTEGRATION CONSTANT

Substituting $t = \frac{1}{\lambda}$ in (14), we get, $N = N_0\, e^{-\lambda.1/\lambda} = N_0\, e^{-1} = \frac{N_0}{e} = \frac{N_0}{2{\cdot}718} = 0{\cdot}368\, N_0 = 36{\cdot}8\%\, N_0$

Thus, the disintegration constant of a radioactive element is the reciprocal of time at the end of which, the number of atoms left undecayed in a radioactive sample reduces to $\left(\frac{1}{e}\right)$ time or 36·8% the original number of atoms (N_0) in the sample.

Units of decay constant λ are s^{-1} or min^{-1} or day^{-1} or $year^{-1}$.

Significance of Decay Constant

The decay constant of a radioactive element determines the rate of decay of the radioactive element. When λ is small, the radioactive substance decays slowly. When value of λ is large, the radioactive substance will decay rapidly.

8(b).19. HALF LIFE OF RADIOACTIVE ELEMENT

Half life of a radioactive element is defined as the time during which half the number of atoms present initially in the sample of the element decay or it is the time during which number of atoms left undecayed in the sample is half the total number of atoms present initially in the sample. It is represented by T.

Expression for Half Life

When $t = T, N = N_0/2$

$$\therefore \text{ From (14), } \quad \frac{N_0}{2} = N_0\, e^{-\lambda T} \quad \text{or} \quad \frac{1}{2} = e^{-\lambda T} \quad i.e.\ 2 = e^{\lambda T}$$

Taking *log* of both sides, we get,

$$\lambda T \log_e e = \log_e 2 = 2{\cdot}3026 \log_{10} 2$$

$$\lambda T \times 1 = 2{\cdot}3026 \times 0{\cdot}3010 = 0{\cdot}6931$$

$$\boxed{T = \frac{0{\cdot}6931}{\lambda}} \qquad ...(15)$$

Thus **half life of a radioactive substance is inversely proportional to the decay constant of the substance.**

Significance of Half Life

Half Life of a radioactive substance measures the time for which the substance lasts for one half of its total number of atoms. If decay constant of radioactive substance is large, its half life is small, and vice-versa.

Table 8(b).2 below lists the half lives of some important radioactive substances. Note that half life of a radioactive substance is not affected by the change in temperature and change in pressure etc.

DO YOU KNOW ?

The half lives of radioactive species show an enormous range of variation. They can be as short as 10^{-15} s and as long as 10^{10} years, estimated age of the universe. The radioactive elements whose half life is short compared to the age of the universe are not found in observable quantities in nature today. They have been identified either in astronomical observations or in nuclear reactions in laboratories.

TABLE 8(b).2.

S. No.	ELEMENT	HALF LIFE
1.	Uranium $_{92}U^{238}$	$4{\cdot}5 \times 10^9$ yrs.
2.	Ionium $_{90}Io^{230}$	$8{\cdot}0 \times 10^4$ yrs.
3.	Radium $_{88}Ra^{226}$	1620 yrs.
4.	Radon $_{86}Rn^{222}$	3·82 days
5.	Polonium $_{84}Po^{210}$	138·3 days
6.	Thorium $_{90}Th^{234}$	24·1 days

Sample Problem **Find the half life of U^{238}, if one gram of it emits $1{\cdot}24 \times 10^4$ α-particles per second. Avogadro's Number = $6{\cdot}025 \times 10^{23}$.**

Sol. Here, atomic wt. of uranium = 238

Avogadro's number = $6{\cdot}025 \times 10^{23}$

Rate of disintegration, $\dfrac{-dN}{dt} = 1{\cdot}24 \times 10^4$

No. of atoms of U^{238} in 1 gram $N = \dfrac{6{\cdot}025 \times 10^{23}}{238}$

According to radioactive decay law, $\dfrac{-dN}{dt} = \lambda N$ or $1{\cdot}24 \times 10^4 = \lambda . \dfrac{6{\cdot}025 \times 10^{23}}{238}$

$$\lambda = \frac{1{\cdot}24 \times 10^4 \times 238}{6{\cdot}025 \times 10^{23}}$$

As half life, $T = \dfrac{0{\cdot}6931}{\lambda} = \dfrac{0{\cdot}6931 \times 6{\cdot}025 \times 10^{23}}{238 \times 1{\cdot}24 \times 10^4}$ sec $= \dfrac{0{\cdot}6931 \times 6{\cdot}025 \times 10^{19}}{238 \times 1{\cdot}24 \times 60 \times 60 \times 24 \times 365}$ years

$= \mathbf{4{\cdot}259 \times 10^{10}}$ **years**

RETAIN IN MEMORY

1. As discussed above, in $t = T$, $N = N_0/2$

In another half life, (*i.e.* after 2 half lives), $N = \dfrac{1}{2}\dfrac{N_0}{2} = \dfrac{N_0}{4} = N_0\left(\dfrac{1}{2}\right)^2$.

After yet another half life, (*i.e.* after 3 half lives), $N = \dfrac{1}{2}\left(\dfrac{N_0}{4}\right) = \dfrac{N_0}{8} = N_0\left(\dfrac{1}{2}\right)^3$

and so on. Hence, after n half lives, $N = N_0\left(\dfrac{1}{2}\right)^n = N_0\left(\dfrac{1}{2}\right)^{t/T}$

where $t = n \times T$ = total time of n half lives.

or
$$\boxed{\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}} \qquad ...(16)$$

2. Some radioactive elements like tritium and plutonium have very small half life. So these elements have decayed long ago. Hence these elements are not found in nature. However, they can be produced artificially by means of nuclear reactions.

3. Number of atoms in excited state, $N_x = N_0\, e^{-(E_x - E_0)/kT}$ where E_0 is energy of atom in ground state, and E_x is energy of atom in excited state.

Sample Problem 1 **The half-life of Radon is 3·8 days. Calculate how much of 15 milligram of Radon will remain after 38 days.**

Sol. Here, $T = 3{\cdot}8$ days, $t = 38$ days, $N_0 = 15$ milligram, $N = ?$

No. of half lives in 38 days is $n = \dfrac{38}{3{\cdot}8} = 10$

We know, $N = N_0\left(\dfrac{1}{2}\right)^n$ $\therefore$ $N = 15\left(\dfrac{1}{2}\right)^{10} = \mathbf{0{\cdot}014\ mg}$

Sample Problem 2 **Half life of Rn^{220} is 55·5 *s*. Calculate decay constant of this element.**

Sol. Here, $T = 55{\cdot}5\ s,\ \lambda = ?$

As $\lambda = \dfrac{0{\cdot}693}{T}$ $\therefore$ $\lambda = \dfrac{0{\cdot}693}{55{\cdot}5} = \mathbf{0{\cdot}0125\ s^{-1}}$

8(b).20. AVERAGE LIFE OR MEAN LIFE OF RADIOACTIVE ELEMENT

A radioactive element undergoes spontaneous disintegration. The process of disintegration follows the laws of probability and continues till infinite time. There are atoms which disintegrate right in the beginning. Their life time or age is zero. There are other atoms which would disintegrate at the end. Their life time or age is obviously infinite. The life time of atoms which disintegrate inbetween, ranges from zero to infinity.

***Average life time of a radioactive element* can, therefore, be obtained by calculating the total life time of all the atoms of the element and dividing it by the total number of atoms present intially in the sample of the element.**

Let us consider a radioactive element containing N_0 atoms to start with *i.e.* at $t = 0$. Let the number of atoms left at time *t* be *N*. Suppose a small number of atoms = *dN* disintegrate further in a small time *dt*. Therefore, the life time of each of these *dN* atoms lies between *t* and $(t + dt)$. If *dt* is taken very small, then the age of each of the *dN* atoms can be taken as *t*.

$\therefore$ Total age of *dN* atoms = *t. dN*

Proceeding in this way, total life time of all the atoms in the sample of the element $= \int_0^{N_0} t.dN$

$\therefore$ By definition, average life time of the radioactive element,

$$\tau = \frac{\text{total life time of all the atoms}}{\text{total no. of atoms}}, \quad i.e., \quad \tau = \frac{\int_0^{N_0} t.dN}{N_0} \qquad \text{...(17)}$$

From (11), $dN = -\lambda N\, dt$

using (14), $dN = -\lambda (N_0 e^{-\lambda t})\, dt$

Substituting in (17), we get, $\tau = \dfrac{\int_\infty^0 -\lambda N_0 e^{-\lambda t} dt \times t}{N_0}$ or $\tau = \lambda \int_0^\infty t.e^{-\lambda t}\, dt$

Note that the limits of integration have been changed in terms of time *i.e.* from (14), when $N = 0$, $t = \infty$ and when $N = N_0$, $t = 0$.

Integrating by parts, we get

$$\tau = \lambda \left[\left\{ t\frac{e^{-\lambda t}}{-\lambda} \right\}_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right] = \lambda \left[0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right] = \int_0^\infty e^{-\lambda t} dt = \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^\infty = 0 - \frac{1}{-\lambda} = \frac{1}{\lambda}$$

$$\therefore \quad \boxed{\tau = \frac{1}{\lambda}} \qquad \text{...(18)}$$

Hence, ***average life of a radioactive element is reciprocal of the decay constant of the element.***

Relation between average life and Half Life

From (15), $\lambda = \dfrac{0{\cdot}6931}{T}$

Putting in (18), we get $\boxed{\tau = \frac{T}{0.6931} = 1.44\,T}$ ...(19)

i.e., the average life of a radioactive element is 1·44 times the half life of the element.

Probability of decay and Probability of Survival

Probability of decay or survival of a nucleus is relevant because the decay process is governed by statistics.

From $-\frac{dN}{dt} = \lambda N$, $\frac{dN}{N} = -\lambda\, dt$

This is the probability of decay in small time dt. Over a large interval of time t, we know

$\boxed{\frac{N}{N_0} = e^{-\lambda t}}$ As N is number of atoms left undecayed.

therefore, this is the **probability of survival.**

The **probability of decay** over this time interval is $\boxed{1 - \frac{N}{N_0} = \frac{N_0 - N}{N_0} = 1 - e^{-\lambda t}}$

Sample Problem **A radioactive isotope X has a half life of 3 seconds. At $t = 0$, a given sample of this isotope contains 8000 atoms. Calculate (i) its decay constant (ii) average life (iii) the time t_1, when 1000 atoms of the isotope X remain in the sample (iv) number of decays/sec in the sample at $t = t_1$ sec.**

Sol. Here, $T = 3$ s, $N_0 = 8000$,

(i) $\lambda = \frac{0.6931}{T} = \frac{0.6931}{3} = \mathbf{0.231\ sec^{-1}}$ *(ii)* $\tau = \frac{1}{\lambda} = \frac{1}{0.231} = \mathbf{4.33\ s}$ *(iii)* $N = 1000,\ t_1 = ?$

As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$ $\therefore$ $\frac{1000}{8000} = \left(\frac{1}{2}\right)^n$ $\therefore$ $n = 3$, $t_1 = n\,T = 3 \times 3 = \mathbf{9\ sec.}$

(iv) $\left(\frac{dN}{dt}\right)_{t=t_1} = \lambda N = 0.231 \times 1000 = \mathbf{231\ s^{-1}}$

8(b).21. ACTIVITY OF A RADIOACTIVE SUBSTANCE

The activity of a radioactive substance is defined as the total rate of disintegration of the substance. It is represented by A.

According to radioactive decay law,

$$N = N_0\, e^{-\lambda t}$$

$$\frac{dN}{dt} = -\lambda N_0\, e^{-\lambda t} = -\lambda N$$

$$\boxed{\text{Activity, } A = -\frac{dN}{dt} = \lambda N} \quad \text{...(i)}$$

If A_0 is activity of the substance at $t = 0$, then $A_0 = \lambda N_0$

$$\frac{A}{A_0} = \frac{\lambda N}{\lambda N_0} = e^{-\lambda t}$$

$$\boxed{A = A_0\, e^{-\lambda t}} \quad \text{...(ii)}$$

Clearly, both A and N decrease exponentially with time.

As $\lambda = \frac{0.6931}{T}$, where T is half life of the substance.

$$\therefore \text{ from } (i), \quad \boxed{A = \frac{0.6931\,N}{T}} \qquad ...(iii)$$

Thus $A \propto \frac{1}{T}$, *i.e.*, greater the half life of the substance, smaller is its activity.

$$\text{Further,} \quad \frac{A}{A_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

Following are the **three units of radioactivity** in common use :

(i) Curie (Ci). The activity of a radio active sample is said to be one curie, when 3·7 × 10¹⁰ decays take place in every one second. Thus

$$\boxed{1\ Ci = 3{\cdot}7 \times 10^{10}\ \text{decays/s.}}$$

This is the approximate activity of 1 gram of radium. In practice, smaller units are used :

1 millicurie (1 $m\,Ci$) = $3{\cdot}7 \times 10^7$ decays/s ;

1 microcurie (1 μ Ci) = $3{\cdot}7 \times 10^4$ decays/s.

(ii) Becquerel (Bq). It is the SI unit of activity.

Thus $\boxed{1\ Bq = 1\ \text{decay/s}}$ $\quad\therefore\quad$ $\boxed{1\ Ci = 3{\cdot}7 \times 10^{10}\ Bq}$

(iii) Rutherford (Rd). The activity of a radioactive sample is said to be one rutherford, when 10⁶ decays take place in every one second. Thus,

$$\boxed{1\ Rd = 10^6\ \text{decays/s.}}$$

Dimensional formula of all the three units of radioactivity $= [M^0\,L^0\,T^{-1}]$.

Sample Problem **A radioactive element decays to 1/32th of its initial activity in 25 days. Calculate its half life.**

Sol. Here, $\frac{A}{A_0} = \frac{1}{32}$, $\quad t = 25$ days

$T = ?$

$$\text{As} \quad \frac{A}{A_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{32} \qquad \therefore \quad n = 5$$

$$\text{As} \quad T = \frac{t}{n} \qquad \therefore \quad T = \frac{25}{5} = \mathbf{5\ days}$$

DO YOU KNOW ?

When we say that activity of a spent reactor fuel rod now is $2{\cdot}7 \times 10^{14}$ Bq, we mean that now $2{\cdot}7 \times 10^{14}$ radioactive nuclei in the rod decay every second. It has no bearing, whatsoever, on the types of radiation emitted, disintegration constants of the nuclei or which nuclei would decay.

DO YOU KNOW ?

1. The level of activity of a sample of a radioactive substance varies directly as the mass of the substance. This level of activity is measured using a device called Geiger Muller counter (GM counter).

2. As $A = A_0\,e^{-\lambda t}$

$$\therefore \quad \frac{dA}{dt} = A_0\,e^{-\lambda t}(-\lambda) = -\lambda A$$

$$= -\lambda(\lambda N)$$

$$= -\lambda^2 N = \frac{-(0{\cdot}693)^2}{T^2} N$$

∴ Instantaneous rate of change of activity varies inversely as the square of half life period of the substance.

8(b).22. ALPHA, BETA AND GAMMA DECAY

The radioactive nuclei are unstable and emit radiations to achieve states of greater stability. The emission of any of the α, β, γ radiations changes the original nucleus, called the ***parent nucleus*** into a new nucleus, called the ***daughter nucleus.**** The daughter nucleus may be radioactive and disintegrate further to form a still newer nucleus. The process continues till a stable nucleus of lead (Z = 82) is formed.

Rutherford and Soddy suggested the following rules governing the radioactive decay.

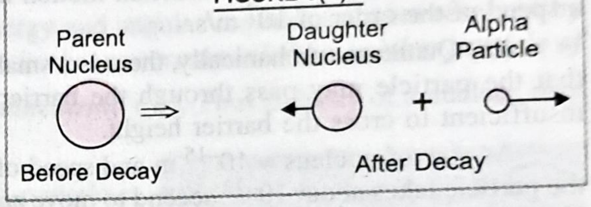
1. *The algebraic sum of charges (atomic numbers) before and after disintegration must be the same.*
2. *The sum of mass numbers before and after disintegration must also be the same.*

Let us now see the three kinds of decay.

8(b).23. ALPHA DECAY

It is the phenomenon of emission of an α particle from a radioactive nucleus.

FIGURE 8(b).8



When a nucleus emits an alpha particle, its mass number decreases by 4 and charge number decreases by 2. For example, when ${}_{92}U^{238}$ undergoes alpha decay, a new element with mass number (238 – 4) = 234 and charge number (92 – 2) = 90 is formed. This corresponds to Thorium. Thus a new element is formed as a result of alpha decay. The transformation is represented by the equation.

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

Note that the alpha decay of ${}_{92}U^{238}$ occurs spontaneously (without any external provocation). The total mass of the decay products ${}_{90}Th^{234}$ and ${}_{2}He^{4}$ is less than the mass of original ${}_{92}U^{238}$. Thus the total mass energy of the decay products is less than the mass energy of the original nuclide. The difference between the initial mass energy and total mass energy of decay products is called ***disintegration energy*** (Q) of the process.

In general, alpha decay is represented as

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q \qquad \text{...(20)}$$

where Q is the energy released in the decay. This can be calculated using Einstein mass energy equivalence relation, $E = (\Delta m).c^2$,

i.e., $$Q = (m_X - m_Y - m_{He})\, c^2 \qquad \text{...(21)}$$

The energy released (Q) is shared by daughter nucleus Y and alpha particle.

We can show that kinetic energy of alpha particle is

$$\boxed{(K.E)_\alpha = \frac{(A-4)\,Q}{A}}$$

As the daughter nucleus is very heavy as compared to the α particle, therefore, most of the disintegration energy (Q) appears as $K.E.$ of α particle.

Explanation of alpha decay

The basic question to be understood is how an α particle, which is bound inside the nucleus on account of nuclear forces, comes out. The potential energy of such an α particle has two components :

(*i*) one component is due to attractive nuclear force-which dominates inside the nucleus making net potential energy negative.

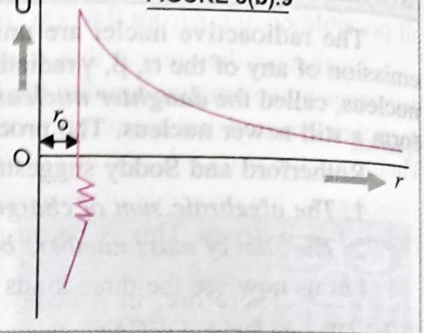
(*ii*) Other component is due to coulomb repulsive force between the α particle and remaining positive charge of the nucleus.

---

*Often the daughter nucleus is more stable than the parent nucleus.

The nuclear component of potential energy of α particle sharply drops as its separation from nucleus increases beyond the radius of the nucleus. The coulomb component of potential energy dominates at this stage. With increasing distance from the nucleus, this component also decreases to negligible values, Fig. 8(b).9.

FIGURE 8(b).9



From various alpha emitters, kinetic energy of α particles emitted is of the order of 4-9 MeV. Studies reveal that nucleus of alpha emitters behaves as a *potential barrier* of height of the order of 25 MeV. *Classically,* it is impossible to account for the emission of an alpha particle of insufficient energy (≈ 4-9 MeV), crossing such a high potential barrier (≈ 25 MeV).

In the year 1928, Gamow, Gurney and Condon explained the *alpha emission in terms of penetration of the nuclear potential barrier on the basis of quantum theory. According to their theory :*

(*i*) An alpha particle may exist as an entity within a heavy nucleus.

(*ii*) The α-particle is in constant motion inside the nucleus with a speed of the order of 10^7 m/s.

(*iii*) Quantum mechanically, there is a small but finite probability that the particle may pass through the barrier even if its energy is insufficient to cross the barrier height.

As size of nucleus ≈ 10^{-15} m and speed of α-particle ≈ 10^7 m/s, the particle takes about 10^{-22} second to move across the nucleus. Thus an alpha particle presents itself 10^{22} times in a second against the potential barrier. As this frequency (ν) is very large, probability P of its escaping the potential barrier is $P = p \times \nu$ where p is probability of escape per collision. As P becomes sufficiently large, the alpha particle may penetrate through the nuclear potential barrier resulting in alpha emission. This is called **tunneling of the nucleus.**

We can now visualise why every $_{92}U^{238}$ nuclide in a sample of $_{92}U^{238}$ atoms does not decay at once. And the half lives for alpha decay of most of the alpha unstable nuclei are very long.

DO YOU KNOW ?

Alpha decay is inhibited by a potential energy barrier that cannot be penetrated according to classical Physics. However, according to quantum mechanics, this potential energy barrier is subject to tunneling. The barrier penetratability and thus the half life for alpha decay are dependent strongly on the energy of emitted alpha particles.

8(b).24. BETA DECAY

Beta decay is the phenomenon of emission of an electron from a radioactive nucleus.

When a parent nucleus emits a β-particle (*i.e.* an electron), mass number remains same because mass of electron is negligibly low. However, the loss of unit negative charge is equivalent to a gain of unit positive charge. Therefore, atomic number is increased by one. For example, when $_{90}Th^{234}$ emits a β-particle, the daughter nucleus has mass number = (234 – 0) = 234 and charge number = (90 + 1) = 91. This is called $_{91}Pa^{234}$. The reaction can be represented as

$$_{90}Th^{234} \longrightarrow {}_{91}Pa^{234} + {}_{-1}e^0 \text{ (β-particle)}$$

In general, we can write

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^0 + Q$$

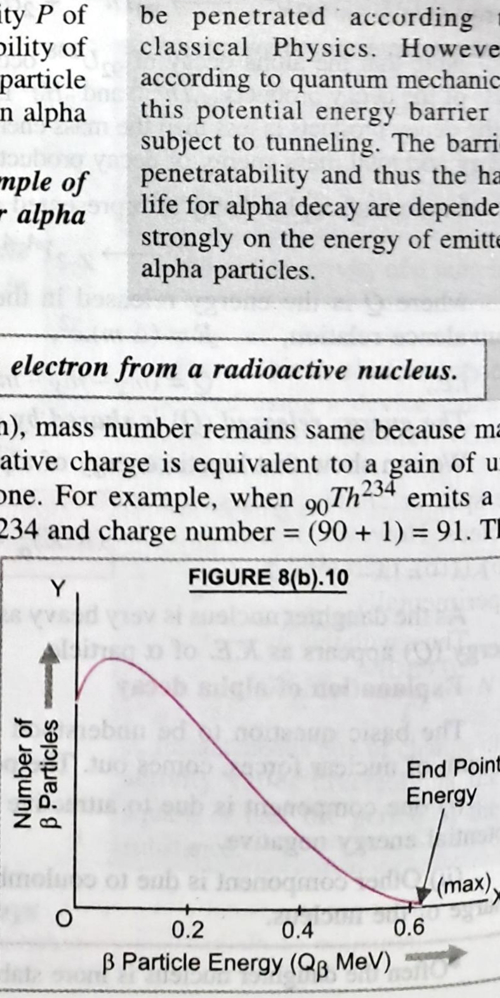
where Q is the energy released in β-decay.

The graph between energy of β-particles and their number is shown in Fig. 8(b).10. From the graph, we find that

(*i*) Most of the β particles emitted carry small energies.

(*ii*) Only very few β-particles carry maximum energy, called **end point energy.**

FIGURE 8(b).10



(iii) The energy spectrum of emitted β-particles is continuous indicating that the β particles can carry all possible energies from 0 to Q_β (max).

Further studies reveal that energy of nucleus emitting β-particles decreases by an amount equal to the end point energy- but most of the β particles emitted have energies smaller than the end point energy. Therefore, the *principle of conservation of energy is violated in β-decay.*

Again, the *principle of conservation of angular momentum is also violated in β-decay.* This is because β particle or electron has a spin $=\frac{1}{2}\left(\frac{h}{2\pi}\right)$. Therefore, in β decay, spin of nucleus must change by $\frac{1}{2}(h/2\pi)$. But in actual practice, there is either *no change* or an integral change in the spin of nucleus emitting β-particle.

DO YOU KNOW ?

In beta decay, the resulting daughter nucleus has the same number of nucleons as the parent nucleus, but it has one less neutron and one more proton.

Neutrino hypothesis

The violations of the principles of conservation of energy and angular momentum were resolved by *Wolfpang Pauli* in the year 1931. He postulated that ***β-decay is always accompanied by another particle of zero rest mass and zero charge. This particle was called antineutrino*** ($\bar{\nu}$). Thus β-decay of a nucleus may be represented as : $_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + Q$

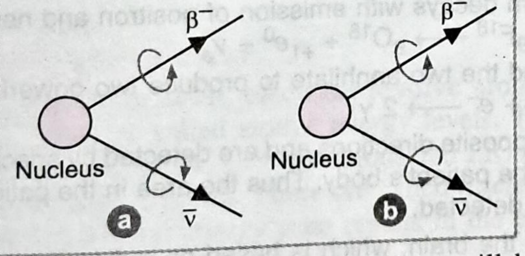
The neutrino hypothesis implies that a low energy β-particle is accompanied by a high energy antineutrino and vice-versa. So the total energy Q of β particle and antineutrino is constant and always equal to end point energy. Thus the law of conservation of energy holds good in β-decay.

Further, antineutrino has a spin $=\frac{1}{2}\left(\frac{h}{2\pi}\right)$. If both, the β particle and antineutrino spin in opposite direction, as shown in Fig. 8(b).11(a), the spin or angular momentum of daughter nucleus will be equal to spin/angular momentum of parent nucleus. However, when both, the β-particle and anti- neutrino spin in the same direction, as shown in Fig. 8(b).11(b), there will be integral change in spin or angular momentum of the daughter nucleus ; as observed experimentally.

DO YOU KNOW ?

Neutrinos are the most abundant particles of Physics. Billions of them pass through our bodies every second leaving no trace whatsoever. Inspite of their elusive character, neutrinos have been detected in the laboratory in 1953 by F. Reines and Cowan. The former was awarded Nobel Prize in Physics in 1995 for this work.

FIGURE 8(b).11



Thus, with this hypotehesis, the law of conservation of angular momentum is also not violated in β-decay.

RETAIN IN MEMORY

1. A positron is similar to an electron in all respects, except that it has a charge + *e*, instead of – *e*.
The symbol β^- is used for an electron and β^+ is used for a positron. Positron is said to be antiparticle of electron.
2. An excited nuclide (which is proton rich) often returns to a stable nuclide by capturing an orbital (K shell) electron, *i.e.* $_ZX^A + {}_{-1}e^0 \longrightarrow {}_{Z-1}Y^A$
This process is called **electron capture** or K-capture. The vacancy caused in K shell is filled by transition of electrons from outer orbits-resulting in the emission of *characteristic X-rays.*

Note that electron capture process is identified by the shell or energy level from which the captured electron comes. If captured electron is from K-shell, it is known as ***K*-shell capture.** If captured electron comes from L-shell, it is called ***L*-shell capture** and so on.

3. For **beta-minus** decay, a neutron transforms into a proton within the nucleus according to :

$$n \rightarrow p + e^- + \bar{\nu}$$

Here, $\bar{\nu}$ represents antineutrino.

And for *beta-plus* decay, a proton transforms into a neutron within the nucleus according to

$$p \rightarrow n + e^+ + \nu$$

As mass of a neutron is greater than the combined mass of a proton and electron, therefore, **Q value of beta minus decay is positive.** The β minus decay process is energetically allowed and the decay may take place in nature spontaneously.

On the other hand, **Q value of beta plus decay is negative.** Therefore, the process of β plus decay is not energetically allowed. It does not take place in nature.

Infact, a free proton is stable. Its life time is greater than the life of our universe. ***A free proton cannot change into a neutron but a free neutron can change into a proton.***

However, inside the nucleus, a proton can change into a neutron and a neutron can change into a proton. This is because inside the nucleus, both protons and neutrons are bound. The *bound nucleons behave differently than free nucleons.* When we take into account the energies of parent and daughter nuclei involved in beta plus decay (or positron decay), the decay is always energetically allowed. For example, ${}_{11}Na^{22} \rightarrow {}_{10}Ne^{22} + e^+ + \nu$

Infact, lighter radioactive isotopes with $Z < 70$ decay mostly by beta plus decay.

CURIOSITY QUESTION

Q. What does PET stand for ? Where is it used ?

Ans. PET stands for *Positron Emission Tomography.* PET scans are used in examining the brain.

A patient is given a dose of FDG (fluoro-deoxy-glucose) which has a fluorine-18 atom attached to a molecule of glucose. The fluorine atom decays with emission of positron and neutrino with a half life of 110 minutes.

$${}_9F^{18} \longrightarrow {}_8O^{18} + {}_{+1}e^0 + \nu_e$$

The positron encounters an electron and the two annhilate to produce two powerful γ-ray photons, *i.e.,*

$$e^+ + e^- \longrightarrow 2\gamma$$

The two photons so produced move in opposite directions and are detected by specialized detectors, which determine their point of origin in the patient's body. Thus the area in the patients body, where glucose metabolism is most intense is detected.

Thus PET is a diagnostic technique for the brain, which is based on radioactive decay.

Sample Problem **After a series of alpha and beta decays, ${}_{94}Pu^{239}$ becomes ${}_{82}Pb^{207}$. How many alpha and beta particles are emitted in the complete decay process ?**

Sol. Here, parent nucleus is ${}_{94}Pu^{239}$ and daughter nucleus is ${}_{82}Pb^{207}$

Decrease in mass number = 239 – 207 = 32

Decrease in charge number = 94 – 82 = 12.

Let number of α particles emitted be x and number of β particles emitted be y.

Now, one alpha decay causes decrease in mass number by 4 and decrease in charge number by 2. Again one beta decay causes increase in charge number by 1 and no change in mass number.

Therefore, $x \times 4 = 32, \quad x = \frac{32}{4} = \mathbf{8}$ and $-y \times 1 + x \times 2 = 12$

$$y = 2x - 12 = 2 \times 8 - 12 = \mathbf{4}$$

8(b).25. GAMMA DECAY

Gamma decay is the phenomenon of emission of gamma ray photon from a radioactive nucleus.

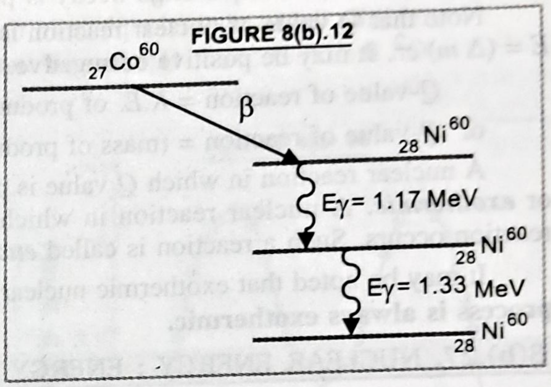
This occurs when an excited nucleus makes a transition to a state of lower energy. As nuclear states have energies of the order of MeV, therefore, the photons emitted by nuclei have very large energies (≈ MeV) and hence much smaller wavelength (< 0·01 A°). Such short wavelength electromagnetic waves emitted by excited nuclei are called γ-rays.

Since photons do not have any charge or rest mass, therefore in γ-decay, daughter nucleus has the same charge number and same mass number as those of parent nucleus. The γ-decay can be represented as

$$_ZX^A \longrightarrow {}_ZX^A + \gamma$$

After an α decay or a β decay, the daughter nucleus is usually in an excited state and it achieves stability by the emission of one or more gamma ray photons.

FIGURE 8(b).12



For example, (*i*) the β-decay of $_{27}Co^{60}$ transforms it into an excited $_{28}Ni^{60}$ nucleus. This reaches the ground state by emission of γ-rays of energy 1·17 MeV and 1·33 MeV. This is shown in Fig. 8(b).12 by an energy level diagram.

The nuclear reactions may be represented as

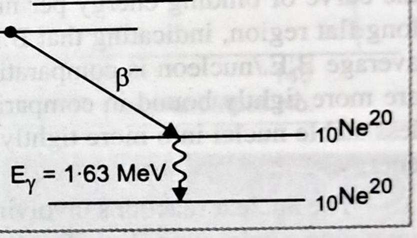
$$_{27}Co^{60} \longrightarrow {}_{28}Ni^{60**} + {}_{-1}e^0 + \bar{\nu}$$

$$_{28}Ni^{60**} \longrightarrow {}_{28}Ni^{60*} + E_\gamma \quad (= 1{\cdot}17 \text{ MeV})$$

$$_{28}Ni^{60*} \longrightarrow {}_{28}Ni^{60} + E_\gamma \quad (= 1{\cdot}33 \text{ MeV})$$

By studying the energies of gamma rays emitted, we can obtain energy levels of the nucleus.

FIGURE 8(b).13



(*ii*) $_9F^{20}$ decays by emitting β⁻ particle to give a daughter nucleus $_{10}Ne^{20}$. This daughter nucleus is in excited state. It comes to ground state by emitting a γ-ray photon of energy 1·63 MeV. This is shown in Fig. 8(b).13 by energy level diagram.

Explanation of gamma decay

It is known that in an atom, electrons revolve around the nucleus in circular orbits, and possess definite energies. These are called atomic energy levels. Transitions between these energy levels account for emission of visible radiations, U.V. radiations and I.R. radiations etc. Studies reveal that similar energy states exist inside the atomic nuclei. These are called ***nuclear energy levels.*** Transition of a nucleus from a higher energy state to a lower energy state results in the emission of energy, which lie in the gamma ray region. This accounts for gamma decay.

8(b).26. NUCLEAR REACTION

A nuclear reaction represents the transformation of one stable nucleus into another nucleus by bombarding the former with suitable high energy particles.

Symbolically, we can represent a nuclear reaction as

$$_ZX^A + {}_2He^4 \longrightarrow {}_{Z+2}C^{A+4} \longrightarrow {}_{Z+1}Y^{A+3} + {}_1H^1 + Q$$

Here, *X* is the *target nucleus*, bombarded by an α particle (called the *projectile*). C is the *compound nucleus* which is unstable. It disintegrates to give *a product nucleus Y* and a *product particle* (proton).

The reaction is named (α, *p*) reaction after the name of the projectile (α)and product particle (*p*). In the nuclear reaction, ***Q* is the *total energy change* in the reaction. It is called nuclear reaction energy or *Q* value of the reaction.**

In all types of nuclear reactions, the ***following conservation laws*** are obeyed : 1. conservation of linear momentum, 2. conservation of total energy, 3. conservation of charge, 4. conservation of number of nucleons.

Rutherford was the first to perform an experiment in 1919 on artificial transmutation of elements. He discovered the nuclear reaction :

$$_7N^{14} + {_2He^4} \longrightarrow {_9F^{18*}} \longrightarrow {_8O^{17}} + {_1H^1}$$

It is (α, p) reaction

Some other examples of nuclear reactions are :

(i) $_3Li^7 + {_1H^1} \longrightarrow {_4Be^{8*}} \longrightarrow {_2He^4} + {_2He^4}$ (p, α) reaction
(ii) $_5B^{11} + {_1H^1} \longrightarrow {_6C^{12*}} \longrightarrow {_6C^{11}} + {_0n^1}$ (p, n) reaction
(iii) $_6C^{12} + {_1H^1} \longrightarrow {_7N^{13*}} \longrightarrow {_7N^{13}} + \gamma$ (p, γ) reaction
(iv) $_7N^{14} + {_0n^1} \longrightarrow {_7N^{15*}} \longrightarrow {_6C^{14}} + {_1H^1}$ (n, p) reaction
(v) $_1H^2 + \gamma \longrightarrow {_1H^1} + {_0n^1}$ (photo disintegration)

Note that **Q value** of nuclear reaction is calculated from Einstein's mass energy equivalence relation, $E = (\Delta m)\, c^2$. It may be positive or negative.

$$Q\text{-value of reaction} = K.E. \text{ of products} - K.E. \text{ of reactants}$$

$$\text{or } Q\text{-value of reaction} = (\text{mass of products} - \text{mass of reactants})\, c^2$$

A nuclear reaction in which Q value is positive, energy is released. Such a reaction is called ***exoergic*** or ***exothermic.*** A nuclear reaction in which Q value is negative, energy has to be supplied before the reaction occurs. Such a reaction is called ***endoergic*** or ***endothermic.***

It may be noted that exothermic nuclear reaction takes place spontaneously. Thus, **radioactive decay process is always exothermic.**

8(b).27. NUCLEAR ENERGY : ENERGY FROM THE NUCLEUS

To have an insight into the process of energy generation from the nucleus, let us examine carefully, the curve of binding energy per nucleon, shown in Fig. 8(b).10. From $A = 30$ to $A = 170$, this curve has a long flat region, indicating that B.E./nucleon in this region is almost constant. For $A < 30$ and $A > 170$, the average B.E./nucleon is comparatively low. It means that in the mid mass region $30 \leq A \leq 170$, the nuclei are more tightly bound in comparison to the nuclei with $A < 30$, and $A > 170$. Therefore, transmutation of less stable nuclei into more tightly bound nuclei would provide an excellent possibility of releasing nuclear energy.

The nuclear reactions involving nuclei with $A > 170$ constitute *nuclear fission.* And the nuclear reactions involving nuclei with $A < 30$ constitute *nuclear fusion.* Thus two distinct ways of obtaining energy from nucleus are (*i*) nuclear fission and (*ii*) nuclear fusion. These processes are discussed below in some detail.

RETAIN IN MEMORY

Nuclear energy is the energy emitted by a nucleus as it becomes more stable.

8(b).28. NUCLEAR FISSION

Nuclear fission is the phenomenon of splitting of a heavy nucleus (usually A > 230) into two or more lighter nuclei.

In the process, certain mass disappears *i.e.* sum of the masses of final products is found to be slightly less than the sum of the masses of the reactant components. This difference in masses is called **mass defect** (Δ*m*). Therefore, as per mass energy relation given by Einstein, energy released in nuclear fission is

$$E = (\Delta m)\, c^2$$

DO YOU KNOW ?

When we get energy from uranium in a nuclear reactor, we are basically rearranging its nucleons into more stable configurations. Similarly, when we get energy from coal by burning it in a furnace, we are rearranging outer electrons of carbon and oxygen into more stable combinations.

In 1938, Hahn and Strassman discovered that when $_{92}U^{235}$ is bombarded with **thermal neutrons,** it splits up into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ with the emission of 3 neutrons alongwith 200 MeV of energy per fission. The neutrons produced after fission are called ***secondary neutrons.*** The reaction is represented as

$$\boxed{{}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\ {}_{0}n^{1} + Q}$$

Here, mass of ${}_{92}U^{235}$ = 235·0439 amu

mass of ${}_{0}n^{1}$ = 1·0087 amu

Total mass of reactants = 236·0526 amu

Again, mass of ${}_{56}Ba^{141}$ = 140·9139 amu

mass of ${}_{36}Kr^{92}$ = 91·8973 amu

mass of 3 ${}_{0}n^{1}$ = 3·0261 amu

Total mass of products = 235·8373 amu

Mass defect (Δm) = Total mass of reactants – Total mass of products

$= 236{\cdot}0526 - 235{\cdot}8373 = 0{\cdot}2153$ amu

As 1 amu = 931 MeV

∴ Energy released per fission of ${}_{92}U^{235}$

$$Q = (\Delta m) \times 931 \text{ MeV} = 0{\cdot}2153 \times 931 \text{ MeV}$$

$$= \mathbf{200{\cdot}4\ MeV}$$

Thus about 200 MeV of energy is released in the fission of a single ${}_{92}U^{235}$ nucleus. This energy appears in the form of γ-rays, kinetic energy of fission fragments and the released neutrons. Further, such a huge amount of energy is released in a very short time ≈ 10^{-9} second.

It is relevant to mention here that the fission fragments of ${}_{92}U^{235}$ are not always Ba and Kr. Other nuclides have also been obtained. But the energy evolved is almost the same in all the cases.

FIGURE 8(b).14

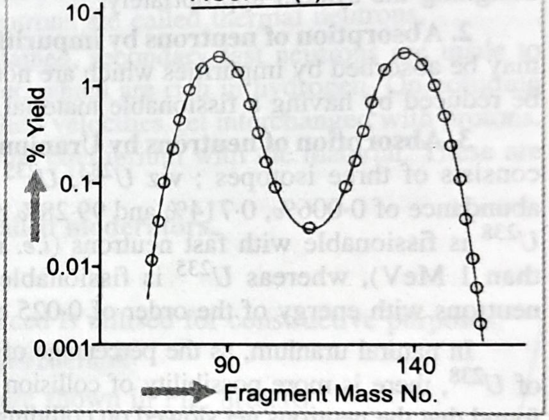


Fig. 8(b).14 represents the distribution by mass number, of the fragments that are found when many fission events of ${}_{92}U^{235}$ are examined. The most probable mass numbers occuring in about 7% of the events are centred around $A \approx 90$ and $A \approx 140$. Note that in Fig. 8(b).14, vertical scale is logarithmic.

Note that the fragment nuclei produced in fission are highly unstable. They are highly radioactive and emit beta particles in succession until each reaches a stable end product.

Also, the disintegration energy in fission events first appears as the kinetic energy of fragments and neutrons. Eventually, it is transferred to the surrounding matter in the form of heat. About 5–6% of energy is associated with neutrinos emitted during β-decay. It is lost to the system.

DO YOU KNOW ?

1 kg of naturally occurring uranium contains about $2{\cdot}56 \times 10^{24}$ atoms of ${}_{92}U^{235}$ isotope.

∴ Total energy released in the fission of 1 kg of naturally occurring uranium

$= 200 \times 2{\cdot}56 \times 10^{24}$ MeV

$= 5{\cdot}12 \times 10^{26} \times 1{\cdot}6 \times 10^{-13}$ J

$\simeq 10^{14}$ J

This is a very huge amount of energy. It can be produced by burning 3 tons of coal.

8(b).29. NUCLEAR CHAIN REACTION

A nuclear chain reaction is said to occur when neutrons emitted from the decay of one nucleus are free to initiate fission in the surrounding nuclei.

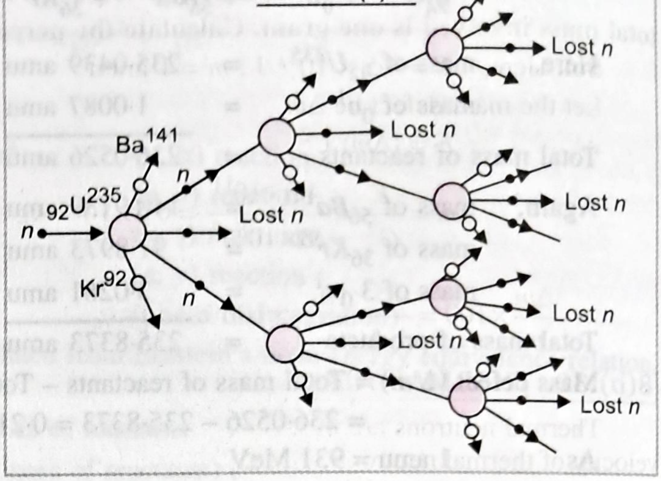
The fission of ${}_{92}U^{235}$ by thermal neutrons is represented as

$$\boxed{{}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\ {}_{0}n^{1} + Q}$$

Now, the three secondary neutrons produced in the reaction may bring about the fission of three more ${}_{92}U^{235}$ nuclei and produce 9 neutrons, which in turn, can bring about the fission of nine ${}_{92}U^{235}$ nuclei and so on. Thus a continuous reaction called ***nuclear chain reaction*** would start and a huge amount of energy will be released in a short time (*i.e.* a few micro seconds). This chain reaction is shown in Fig. 8(b).15.

Here, FP represents the fission product. If we can have a control over this reaction, the energy released can be utilised for peaceful purposes. This led to the construction of ***Nuclear Reactor or Nuclear Pile.*** However, if the reaction cannot be controlled, the energy released will bring about disastrous effects. This led to the discovery of an ***atom bomb***.

FIGURE 8(b).15



The following points should be considered for a ***self propagating nuclear chain reaction.***

1. Leakage of neutrons from the system. Some of the secondary neutrons produced may escape out of the system and will not take part in further fission. This leakage may be reduced by designing the system appropriately.

2. Absorption of neutrons by impurities. The secondary neutrons may be absorbed by impurities which are not fissionable. This loss may be reduced by having a fissionable material free from impurities.

3. Absorption of neutrons by Uranium–238. The natural uranium consists of three isotopes ; viz U^{233}, U^{235} and U^{238}, having relative abundance of 0·006%, 0·714% and 99·28% respectively. It is found that U^{238} is fissionable with fast neutrons (*i.e.* neutrons with energy more than 1 MeV), whereas U^{235} is fissionable with slow neutrons (*i.e.*, neutrons with energy of the order of 0·025 eV).

In natural uranium, as the percentage of U^{235} is much less than that of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on colliding with U^{238}. As a result of it, further fission of U^{238} is not possible. That is why *the chain reaction in natural uranium cannot occur.*

DO YOU KNOW ?

For nuclear power plants, U^{235} should be 1–4% and for nuclear bombs, U^{235} must be closer to 97%. The process of increasing the percentage of U^{235} in a sample of uranium is called **Enrichment of Uranium.**

To sustain nuclear chain reaction, the percentage of $_{92}U^{235}$ isotope in the uranium ore is increased from 0·7% to 3%. $_{92}U^{235}$ content of increased percentage, *i.e.*, 3% is known as **Enriched Uranium.**

4. Critical size. In order to have a sustained chain reaction in a sample of U^{235}, it is required that the number of neutrons lost due to leakage and absorption should be much smaller than the number of neutrons produced in a fission process. This is possible if the uranium block has a size greater than a certain critical value. It is called **critical size.**

The chain reaction once started will remain steady, accelerate or retard depending upon, what is called the ***neutron reproduction factor*** or ***multiplication factor. This is the ratio of rate of production of neutrons to the rate of loss of neutrons due to leakage and absorption i.e.***

$$\text{Reproduction factor, } K = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

If $K = 1$, the chain reaction will be steady or sustained. The size of the fissionable material used is said to be the **criticial size** and its mass, the **critical mass.**

This is the stage required for steady power generation, when criticality is said to have been reached.

If $K > 1$, the chain reaction accelerates resulting in an explosion. The stage is said to be **Super critical.** This is how an **atom bomb** works.

If $K < 1$, the chain reaction gradually comes to a halt. The stage is said to be **subcritical.**

DO YOU KNOW ?

A thermal neutron with relatively small kinetic energy can penetrate a nucleus, whereas a proton or an alpha particle would need a much larger amount of energy to penetrate the same nucleus. This is primarily because proton or alpha being positively charged have to overcome the repulsive forces due to protons in the nucleus.

Sample Problem **The energy released in a nuclear fission process is $4{\cdot}5 \times 10^{11}$ J. Where in total mass involved is one gram. Calculate the percentage of mass defect in the process.**

Sol. Here, $E = 4{\cdot}5 \times 10^{11}$ J ; $m = 1$ gram

Let the mass defect be Δm

$$\therefore \quad E = (\Delta m)\, C^2$$

$$\Delta m = \frac{E}{C^2} = \frac{4{\cdot}5\times10^{11}}{(3\times10^8)^2} = 0{\cdot}5\times10^{-5}\ \text{kg}$$

$$\frac{\Delta m}{m}\times100 = \frac{0{\cdot}5\times10^{-5}}{10^{-3}}\times100\% = \mathbf{0{\cdot}5\%}$$

8(b).30. THERMAL NEUTRONS

Thermal neutrons are the low energy neutrons or slow moving neutrons. Their energy ≈ 1/40 eV. The velocity of thermal neutrons ≈ 2·2 km/s, which corresponds to the velocities of random motion of atoms and molecules in a gas at room temperature. That is why these neutrons are called thermal neutrons.

To obtain thermal neutrons so that chain reaction is sustained, secondary fast neutrons are made to pass through substances like paraffin, deuterium or heavy water, which are rich in hydrogen. On colliding against the hydrogen nuclei or protons of roughly equal mass, their velocities get interchanged with protons. The neutrons are thus slowed down, and are therefore in thermal equilibrium with the material. These are thermal neutrons.

The materials used for slowing down fast neutrons are called **moderators.**

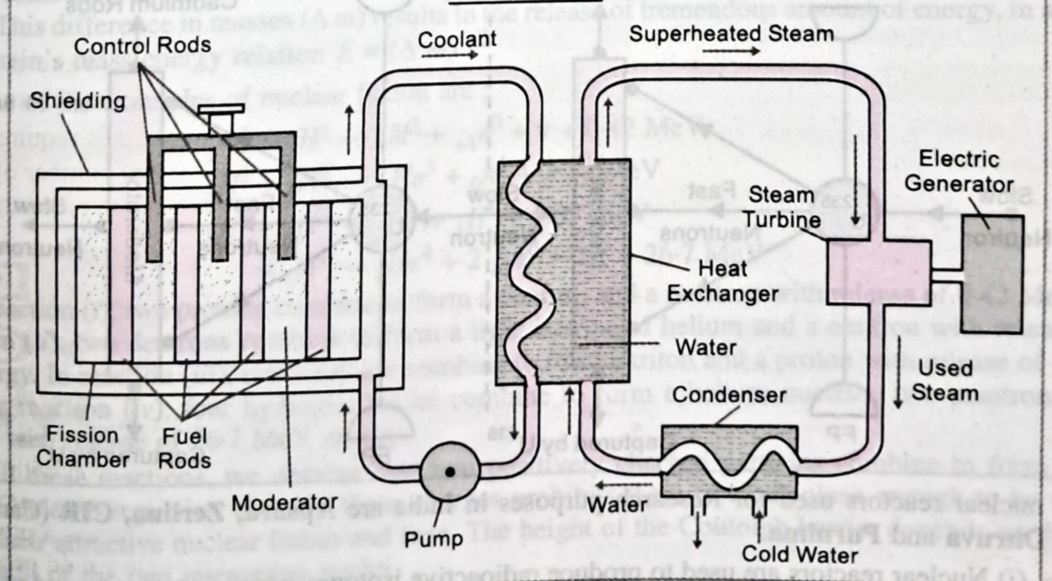
8(b).31. NUCLEAR REACTOR

It is a powerful device, wherein the nuclear energy produced is utilised for constructive purposes.

A nuclear reactor is based upon ***controlled nuclear chain reaction.***

Construction. The main components of nuclear reactor are shown in Fig. 8(b).16.

FIGURE 8(b).16



(1) **Nuclear Fuel.** *It is a fissionable material to be used for the fission process to take place.* Commonly used fuels in a nuclear reactor are U^{233}, U^{235}, Pu^{239} etc. Generally, uranium oxide pellets are inserted end to end into long hollow metal tubes constituting the fuel rods. When slow neutrons interact with the fuel, the fission starts and the energy is released.

(2) Moderator. *Its function is to slow down the fast moving secondary neutrons produced during the fission.* The material of moderator should be light and it should not absorb neutrons. Usually, heavy water, graphite, deuterium and paraffin etc. can act as moderators. These moderators are rich in protons.

When fast moving neutrons collide head on with the protons of moderator substances, their energies are interchanged and thus the neutrons are slowed down. Such neutrons are called *thermal neutrons* which cause fission of U^{235} in the fuel.

(3) Control rods. *They have the ability to capture the slow neutrons.* To control the chain reaction from becoming violent, rods of boron or cadmium (called control rods) are inserted in the holes of reactor core, up to a desirable length. As a result of it, the desired number of neutrons are absorbed and only limited number of neutrons are left to produce fission. These rods can be adjusted from outside the reactor, in order to control the chain reaction.

In addition to control rods, reactors are provided with *safety rods*, which when required, can be inserted into the reactor. They reduce the neutron reproduction factor (K) to less than unity.

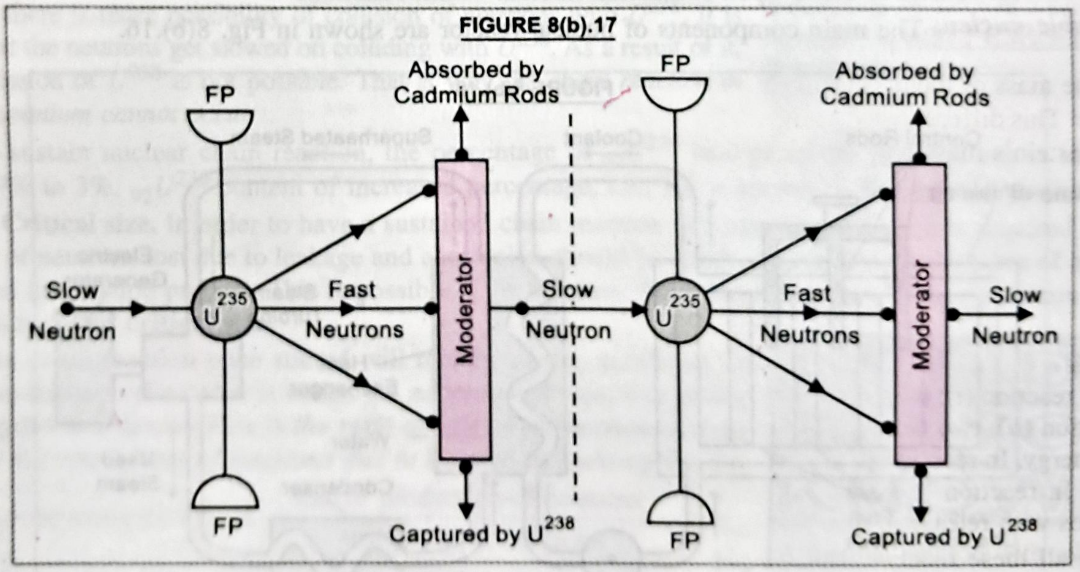
(4) Coolant. *A substance which is used to remove the heat produced and transfer it from the core of the nuclear reactor to the surroundings is called coolant.* At ordinary temperatures, water and heavy water serve as coolants but at high temperatures, generally liquid sodium is used as a coolant.

The *coolant takes up the heat energy produced in nuclear fission and passes on this energy to water in a heat exchanger.* As a result of it, superheated steam is produced which drives a turbine coupled with an electric generator, [Fig. 8(b).16.]

(5) Shielding. The whole reactor is protected with concrete walls, 2 to 2.5 metre thick, so that radiations emitted during nuclear reactions may not produce harmful effects on the persons working on the reactor.

Working. Fig. 8(b).17 outlines the working of nuclear reactor. Slow neutrons cause the fission of U^{235} nuclei. FP stands for fission product. To start the nuclear reaction, the cadmium rods are slowly removed and to stop it, they are inserted. This brings out a controlled nuclear chain reaction and hence the energy produced can be used for constructive purposes.

FIGURE 8(b).17



The nuclear reactors used for Research purposes in India are **Apsara, Zerlina, CIR** (Canada India Reactor), **Dhruva** and **Purnima.**

Uses. (*i*) Nuclear reactors are used to produce radioactive isotopes which in turn are used in *medicine, industry* and *agriculture*.

(*ii*) They are used in electric *power generation.*

Four nuclear power plants in India are :

(*a*) Tarapur Atomic Power Station in Maharashtra

(b) Rajasthan Atomic Power Station near Kota

(c) Narora Atomic Power Station in U.P.

(d) Kalpakkam Atomic Power Station in Tamil Nadu.

(iii) They can be used for the *propulsion of ships*, submarines and air crafts.

(iv) They are used to produce *neutron beam* of high intensity which is used in the *treatment of cancer* and *nuclear research*.

Fast Breeder Reactors. They use thorium or natural uranium as fuel elements. When fast neutrons strike uranium fuel in a reactor, ${}_{92}U^{238}$ (which is present along with ${}_{92}U^{235}$ in natural uranium) absorbs a neutron and becomes ${}_{92}U^{239}$. This is radioactive and undergoes β-decay twice to produce ${}_{94}Pu^{239}$, as represented here : ${}_{92}U^{238} + {}_0n^1 \longrightarrow {}_{92}U^{239} \xrightarrow{-\beta} {}_{93}Np^{239} \xrightarrow{-\beta} {}_{94}Pu^{239} \xrightarrow{\alpha} {}_{92}U^{235}$

Now ${}_{94}Pu^{239}$ is fissionable and undergoes alpha decay (with $T = 24000$ years) to produce ${}_{92}U^{235}$. Thus using an unfissionable material ${}_{92}U^{238}$, we produce fissionable material ${}_{94}Pu^{239}$, which can be fissioned even by fast neutrons. Therefore, no moderator is used in such reactors.

RETAIN IN MEMORY

1. Note that in an atom bomb, nuclear chain reaction is uncontrolled, whereas in a nuclear reactor, nuclear chain reaction is controlled.
2. An unavoidable feature of reactor operation is the accumulation of radioactive waste, including both, fission products and heavy transuranic elements such as plutonium and americium. Thus the waste of a nuclear power station is extremely hazardous to all forms of life on earth. Elaborate safety measures are needed not only for reactor operation, but also for handling and disposal of **radioactive waste.**

8(b).32. NUCLEAR FUSION

***Nuclear fusion* is the phenomenon of fusing two or more lighter nuclei to form a heavier more stable nucleus.**

The mass of the product nucleus is slightly less than the sum of the masses of the lighter nuclei fusing together. This difference in masses (Δm) results in the release of tremendous amount of energy, in accordance with Einstein's mass energy relation $E = (\Delta m)\, c^2$.

Some of the examples of nuclear fusion are :

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_{+1}e^0 + \nu + 0{\cdot}42 \text{ MeV} \quad \text{...(i)}$$

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1 + 3{\cdot}27 \text{ MeV} \quad \text{...(ii)}$$

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4{\cdot}03 \text{ MeV} \quad \text{...(iii)}$$

e.g.,

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\,\nu + 26{\cdot}7 \text{ MeV} \quad \text{...(iv)}$$

In reaction (*i*), two protons combine to form a deutron and a positron with release of 0·42 MeV energy. In reaction (*ii*), two deutrons combine to form a light isotope of helium and a neutron with release of 3·27 MeV energy. In reaction (*iii*), two deutrons combine to form a triton and a proton with release of 4·03 MeV energy. In reaction (*iv*), four hydrogen nuclei combine to form a helium nucleus, two positrons and two neutrinos with release of 26·7 MeV energy.

In all these reactions, we observe that two positively charged particles combine to form a heavier nucleus. Coulomb repulsion between these charges prohibits them to come close enough to be within the range of their attractive nuclear forces and fuse. The height of the Coulomb barrier depends on the charges and the radii of the two interacting nuclei.

The essential condition for carrying out nuclear fusion is to raise the temperature of the material so that particles have enough energy due to their thermal motions alone and they can penetrate the Coulomb barrier.This process is called **Thermonuclear fusion.**

Thus for thermonuclear fusion to occur, extreme conditions of temperature and pressure are required. Further, higher density is also desirable so that collisions between light nuclei occur quite frequently. These conditions are met only in the interior of stars, and cannot be arranged in a laboratory. The energy generation in sun and stars takes place via thermonuclear fusion.

8(b).33. DISTINCTION BETWEEN NUCLEAR FISSION AND NUCLEAR FUSION

As detailed above, we find that

(*i*) Both, nuclear fission and nucler fusion are the sources of tremendous energy.

(*ii*) In both the processes, a certain mass (Δm) disappears, which appears in the form of energy as per Einstein equation : $E = (\Delta m)\, c^2$

(*iii*) In fission, a heavy nucleus splits into two or more lighter nuclei. In fusion, two or more lighter nuclei fuse together to form a heavier nucleus.

(*iv*) For carrying out fission, a suitable bullet or projectile like neutron is needed. For carrying out fusion, the lighter nuclei have to be brought very close to eachother against electrostatic repulsion. For this, suitable energy is to be made available, often by raising the temperature to the order of 10^7 K. This justifies nuclear fusion being called *thermonuclear fusion.* In actual practice, such high temperatures are generated by nuclear fission. That is why usually, ***nuclear fission precedes nuclear fusion.***

(*v*) In a nuclear fusion reaction, energy liberated per unit mass of their nuclei is many times larger than the energy liberated per unit mass in nuclear fission reaction. Therefore, for a given weight, hydrogen bomb (based on nuclear fusion) is far more dangerous than an atom bomb (based on nuclear fission).

(*vi*) The products of nuclear fission reaction are radioactive. They produce environmental pollution and hence require very careful disposal. However, the products of nuclear fusion are not radioactive. They are harmless and can be disposed off easily.

(*vii*) While producing nuclear energy from fission, we have learnt how to control nuclear chain reaction. But we have yet to learn controlling the thermo nuclear fusion reactions. This would be the basis of *fusion reactor*, which is seen as the future source of unlimited energy without pollution.

Sample Problem **The mass defect in a nuclear fusion reaction is 0·3%. What amount of energy will be liberated in one kg fusion reaction.**

Sol. Mass defect, $\Delta m = 0{\cdot}3\%$ of 1 kg $= \dfrac{0{\cdot}3}{100} \times 1\,\text{kg} = 3 \times 10^{-3}$ kg

Energy liberated $= (\Delta m)\, c^2 = 3 \times 10^{-3}\,(3 \times 10^8)^2 = \mathbf{2{\cdot}7 \times 10^{14}}$ **joule**

8(b).34 CONTROLLED THERMONUCLEAR FUSION

Controlled thermonuclear fusion is the basis of fusion reactor, which is the future source of unlimited and unpolluted energy.

The most attractive reactions for terrestrial use are :

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1 + 3{\cdot}27 \text{ MeV}$$

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4{\cdot}03 \text{ MeV}$$

$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1 + 17{\cdot}59 \text{ MeV}$$

Deuterium, the source of deutrons for these reactions is available in unlimited quantity in *sea water.* Other requirements for a fusion reactor are :

(*i*) ***High particle density.*** The deutron-deutron collision rate can be high only when the density (number) of interacting particles is very large. At high temps required, deuterium will be completely ionised, forming neutral plasma.

(*ii*) ***A high plasma temperature*** of the order of 10^9 K is required so that interacting particles can penetrate the Coulomb barrier and fuse together.

(*iii*) ***A long confinement time.*** The hot plasma must be maintained at a sufficiently high density and temperature for a sufficiently long time so that fusion of enough fuel occurs. No solid container can withstand these conditions. Therefore, clever confining techniques, such as magnetic confinement and inertial confinement are being explored.

Efforts are being made all the world over to achieve controlled thermonuclear fusion in the laboratory.

8(b).35 NUCLEAR HOLOCAUST

Nuclear holocaust ***is the name given to large scale destruction and devastation that would be caused by the use of nuclear weapons.***

The devastating damage caused by nuclear weapons can be classified as :

(*i*) Immediate Effects (*ii*) Long term Effects

Immediate Effects include *thermal flash* causing spontaneous ignition and setting fires ; *shock waves* which are high pressure waves moving out at speeds greater than 3000 km h^{-1} ; *Electromagnetic pulse* capable of destroying power distribution systems, telecommunications and computer networks and ***Initial Nuclear Radiation*** which may cause instant death of people and animals.

Long Term Effects include *Radioactive Fall out* especially from small fission bombs and *nuclear winter* that may follow large scale detonations. Sunlight might be blocked from the earth's surface resulting in lower temperatures and destruction of plant life. The protective ozone layer could also be damaged.

8(b).36 INDIA'S ATOMIC ENERGY PROGRAMME

The Atomic Energy Programme of our country was launched around 1950 under the leadership of Homi J. Bhabha. The major milestones achieved so far are :

(*i*) First nuclear reactor named Apsara went critical on August 4, 1956. It used enriched uranium as fuel and water as moderator.

(*ii*) Another reactor named Canada India Reactor (CIRUS) became operative in 1960. It used natural uranium as fuel and heavy water as moderator.

(*iii*) Indigenous design and construction of plutonium plant at Trombay. It ushered in the technology of fuel reprocessing.

(*iv*) Research reactors like Zerlina, Purnima, Dhruva and Kamini were commissioned. The last one uses *U*–233 as fuel.

(*v*) The fast breeder reactors which use Plutonium–239 as fuel do not need moderators. They can be used to produce fissile Uranium–233 from Thorium–232 and to build power reactors based on them. Considerable work has been done by our scientists in this direction.

(*vi*) We have mastered the complex technologies of mineral exploration, mining, fuel fabrication, heavy water production, fuel reprocessing etc.

Elaborate safety measures, both in the design and operation of reactors and stringest standards of radiological protection are most important hall marks of Indian Atomic Energy Programme.